Single and Multiple Glider Path Planning using an Optimization-based Approach

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***Abstract*— Path planning for Unmanned Underwater Vehicles (UUVs) is a key issue for the success and efficiency of the missions these vehicles perform. This problem is very challenging, because it must cope with dynamic and uncertain models both for the vehicle and for the environment. In the case of ocean gliders, this aspect is critical due to the strong influence of ocean currents on the glider navigation.**

In this paper, we present a novel path planning scheme for gliders based on iterative optimization that shows promising results on realistic simulations, including highly time-dependent ocean currents. This method models the glider as an intelligent agent that senses the ocean currents speed and direction, and generates an path according to the predefined objectives. The method can be easily configured and adapted to various optimization problems. Here, we include an example of coordinated path planning, in which the paths of a fleet of gliders is optimized, subject to constraints. Also, the proposal reflects accurately the physical vehicle navigation and gives a superior performance when compared with other approaches.

I. Introduction

Throughout the last two decades, Unmanned Underwater Vehicles (UUVs) have proven a worthy tool for a wide range of applications. They encompasses but are not limited to structure inspection, environmental monitoring and control or security. In the ocean, gliders are particularly suitable for these tasks, because of their peculiar energy-saving navigation system. It endows them with a large autonomy, at the expense of moving relatively slow. For this reason, they can stay for long periods of time in water, but they drift substantially by the action of ocean currents. This issue clearly motivates the demand of path planners to conduct such missions. Path planning makes a difference by means of offering intelligent paths that save time and power. However, we are facing a challenging problem, due to the dynamism and uncertainty of the ocean and its high impact on the glider.

We distinguish two clearly differentiated stages in the glider operation, as figures 1 and 2 show. In a first stage, the glider is on surface, so it can communicate with the base station to transmit data or receive control commands, such as the heading. During this surfacing interval, the vehicle knows its location using the GPS. Once it dives the second stages begins, during which it is not possible to communicate, neither know the location with the GPS. Thus, it is only possible to estimate the location with some uncertainty, as Fig. 2 shows. At this stage, mission data is acquired while the glider navigates describing a yo-yo profile by means of changing its buoyancy (see Fig. 1). This produces a vertical impulsion that is transformed into an effective but low surge speed thanks to internal mass displacements and the wings and tail orientation. Thereby, we obtain the characteristic climb/dive transects of Fig. 1. Additionally, ocean gliders estimate ocean currents while they are adrift on surface. The glider control algorithm uses this information to correct the actual bearing. However, this local correction cannot compete with global path planning techniques in most ocean environments.

In terms of power consumption, the glider yo-yo profile is very efficient. Besides processing and communication, the batteries are only used intensively during a small fraction of the cycle time to change the vehicle buoyancy, using an electric pump; and, much less demanding, to modify the vehicle attitude and bearing angle while submersed using low consumption actuators. Not surprisingly, ocean gliders have been extensively applied in Maritime Research, and they are expected to become one of the reference technologies as observational tool in the coming years [1].

In the following section we discuss the state-of-the-art of path planning algorithms for ocean gliders. We make a thorough analysis of different approaches and their applicability to the problem at hand. Subsequently, in Sec. III we motivate the interest of path planning for glider in time-dependent environments, as well as our proposal. Then, we introduce our approach in Sec. IV, and the experimental results are discussed later in Sec. V. Finally, section Sec. VI enumerates the conclusions derived from this work.

II. Review of Related Works

Planning for UUVs has been a subject of great interest for researchers since the introduction of these robotic platforms. Several approaches have been developed applying techniques that comprise searching algorithms that come from areas like Artificial Intelligence, potential field modeling, Evolutionary Computing, probabilistic sampling-based methods, Optimal Control, and so on. Some of the most relevant, in our honest opinion, are described in the following.

However, before examining those techniques, we think it is worth noting that ocean gliders drift from the expected trajectory as a direct consequence of ocean currents. This makes a strong difference with classical ground robots, for

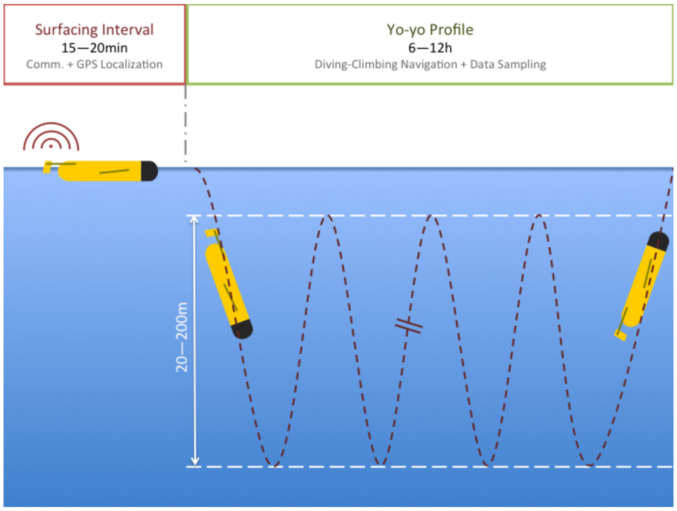

Fig. 1

THE GLIDER OPERATION CYCLE IS COMPOSED OF TWO DIFFERENTIATED PHASES: THE SURFACING INTERVAL AND THE YO-YO PROFILE STINT WHILE SUBMERSED. IN BRIEF, DURING THE SURFACING INTERVAL THE GLIDER COMMUNICATES THE DATA COLLECTED IN THE PREVIOUS STINT, LOCALIZES ITSELF WITH THE GPS AND PROBABLY RECEIVE THE NEXT BEARING/WAYPOINT. WHILE SUBMERSED IT NAVIGATES DESCRIBING A YO-YO PROFILE WITHIN A RANGE OF DEPTHS AND COLLECTS MISSION DATA.

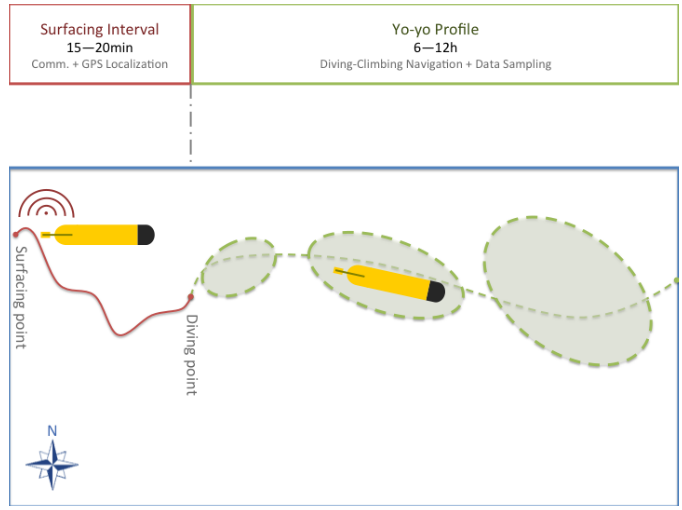

Fig. 2

THIS TOP VIEW OF THE SURFING INTERVAL AND THE YO-YO PROFILE STINT SHOWS HOW ON SURFACE THE GLIDER TRAJECTORY IS KNOWN USING THE GPS, BUT WHILE SUBMERGED AFTER THE DIVING POINT IT IS UNKNOWN, ALTHOUGH IT CAN BE ESTIMATED UP TO SOME UNCERTAINTY. THE MAIN SOURCE OF UNCERTAINTY IS THE DRIFTING CAUSED BY OCEAN CURRENTS. AT EACH SURFACING POINT SUCH UNCERTAINTY COLLAPSES WITH THE FIRST GPS FIX.

which there is a number of path planning methods. Regarding path planning, the current field conforms an anisotropic and dynamic cost map. Also, gliders have, at least, one Degree of Freedom (DoF), being the bearing, which can only be modified at discrete times, when they are on surface.

Graph methods represent the search space with a grid-shape graph, where the edges are labeled with the cost of traveling from a vertex to one of its neighbors. Most of these methods come from the field of Artificial Intelligence, being A* [2] probably the first graph method adapted to the problem at hand [3], with the strong assumption of static ocean currents. Note that some authors use Wavefront Expansion (WE) [4], [5], which does not use any heuristic and therefore it is identical to Dijkstra's algorithm [6]. Anyway, the approach is still the same, basically. To alleviate the computational cost, some authors represents the search space with adaptive grids. For example, Carroll et al. [7] apply this strategy on a quad-tree search space. Our implementation of A*, included in this paper for comparison purposes, is equivalent to Garau's [3].

The approaches based on the minimization of energy functions are also worth commenting. As good examples, we can cite the work of Kruger et al. [8], that includes the time as an extra dimension in the search space, or Witt et al. [9], that incorporate modeling of time-varying obstacles using potential fields. The main drawback of potential fields is that they often get trapped in local minimum.

Sampling-based methods try to overcome the combinatorial explosion of the optimal path planning search to some extend. Rapidly-exploring Random Trees (RRT) [10], [11] constitute an incremental sampling and searching approach that has been applied to both AUVs [12] and gliders [13]. The main drawback of this method is the inability to handle time-dependent ocean currents. Furthermore, as a consequence of the probabilistic nature of sampling-based methods, optimality is no longer assured. Thus, the paths found often require further refinement, which is not straightforward in the problem at hand.

The problem of local minimum has been tackled by means of strategies based on particle swarms, simulated annealing, or genetic algorithms, all of them techniques from Evolutionary Computing. Genetic algorithms are used in [14] for AUVs in the presence of time-dependent ocean currents. Execution time limitations in the population size and the number of generations, reduce the optimality of the path found. Simulated annealing is a global optimization technique that is usually applied as a post-processing refinement over previously generated results. However, it is very sensitive to the initial guess, which must be selected wisely. For this reason it is common to find it integrated as part of other methods, e.g. a swarm optimization in [9].

Optimal paths can be obtained solving a Boundary Value Problem (BVP), for the Optimal Control specification of the problem. In [15] the glider is modeled as a Dubins car, using Dubins curves for steady flow and turn constraints, and Zermelo's optimal navigation formula for unsteady flow. In either case, these are local planning methods that perform poorly with strong currents, so only propelled vehicles might benefit from them. Also, there is a number of works that use Lyapunov Exponents to analyse ocean currents stability. This approach produces Lagrangian Coherent Structures (LCS) that can simplify path planning to some extend. This is the case of the non-linear trajectory generation or NTG method [16] applied over B-Splines of Zhang et al. [17].

Also, some authors have envisioned a number of approaches that combined some of the methods discussed thus far. Eichhorn in [18] proposes a combination of A* and Zermelo's optimal navigation formula to reduce the computational cost of the algorithm. In a previous work, we developed an algorithm the combines A* a sampling strategy [19], termed Constant-Time Surfacing A* (CTS-A*), which is included in the experimental results for comparison. Also, Soulignac's approach [5] incorporates local optimization for the running path found with the Wavefront Expansion algorithm.

In the literature of ocean glider piloting and planning there is a number of works that addresses additional optimization objectives and applications. In [20] the objective function is a weighted sum of the travel time and energy consumption. The Hold-Track problem introduced in [18], that consists on finding the path with the lowest deviation from an user-defined trajectory, but still reaching the target waypoint. The optimal departure-time is covered in [21] with a Symbolic Wavefront Expansion algorithm. Evolving features, such as the centroid or boundary points of a bloom, are tracked in [22]. Most of these problems are well accommodated by our proposal.

Finally, a line that has received recently a lot of attention from researchers is the use of multiple vehicles in a coordinated mission. Some relevant examples include [23] and [24], that face the problem of adaptive sampling of oceanic variables by means of gliders fleets.

III. Motivation

In this work, we introduce a novel path planning method to optimize the path for a glider subjected to time-dependent ocean currents, that bases on iterative optimization. In the optimization process the glider navigations model is followed faithfully. Our method exploits the temporal discretization of the stints, i.e. the fact that the yo-yo profile lasts a constant time and the bearing is constant too. Therefore, we manage a continuous representation of the bearing space, so the problems discussed thus far are diminished.

In this paper, we analyse two different problems, both for a single glider and a fleet of gliders travelling in parallel trajectories. First, the problem of finding a path such that, after travelling for a fixed time, the glider is left at the closest possible distance to the target waypoint. Here, we extend the work presented in [25], by means of analyzing the influence ocean currents direction on the paths found for this problem. Second, we try to find the path of minimum temporal cost that gets a glider moving at constant speed on a time-dependent ocean current field from a starting point to a target point. We have termed them the distance-based and time-based problems, respectively. In either case, the glider bearing is configurable (1 DoF), but only in discrete instants of time. It is worth mention that on the basis of those problems it is possible to solve more complex or elaborated ones. Also, the method is quite flexible, as it can be applied to a number of other optimization problems with few adaptation or configuration.

Similar in spirit to our work, Moqin et al. [26] propose an iterative optimization process for glider planning. However, the focus of this work is centered on enhance waypoint precision, and not in optimal path planning generation. Besides, only static current maps are considered. In the next section we explain our proposal in detail, and then we try to show that it outperforms other approaches: the computational cost is lower than graph methods when these are forced to find the optimum, the paths obtained are always better than those found with sampling-based methods, and it is also applicable in a wide range of problems where other techniques, that impose some assumptions, do not.

IV. Iterative Optimization-based Path Planner

First of all, we consider some assumptions that are also commonly imposed in other works. Only surface ocean currents are contemplated, normally computing vertical means. Hence, a simple glider motion model with a single DoF is used, i.e. it is modeled as a moving particle with constant speed and configurable bearing, under the influence of ocean currents.

The core of our method optimizes the glider bearings and applies a simple kinematic model of the glider to simulate its trajectory. With the election of the bearings as optimization variables the benefit is twofold. We avoid the spatial discretization and allow for a physically realistic simulation. Classical Levenberg-Marquardt, Sequential Quadratic Programming or Quasi-Newton methods have been tested.

The objective function is designed on the basis of a stint simulator that reproduces the glider behaviour by combining the commanded bearing with the current model data and the nominal glider velocity. As commented previously, due to the relative low surge speed, the resultant glider trajectory is strongly influenced by ocean currents. This effect can be observed in Fig. 3.

A. The Algorithm

The time-based problem can be formulated as a direct optimization process. In this scenario the identification of the optimization parameters is known in advance, as they become a direct function of the stint duration and the path planning horizon. For example, the Slocum electric gliders are normally programmed for $8\mathrm{h}$ stints, so the optimization process for 3 days will take 9 bearings as parameters. The initialization phase can use either the start to target angle for each bearing or the result of a simple approach, like the Direct to goal simulation, which will be introduced later.

The distance-based problem requires a more sophisticated approach, because the number of optimization parameters for a direct solution cannot be computed in advance. On

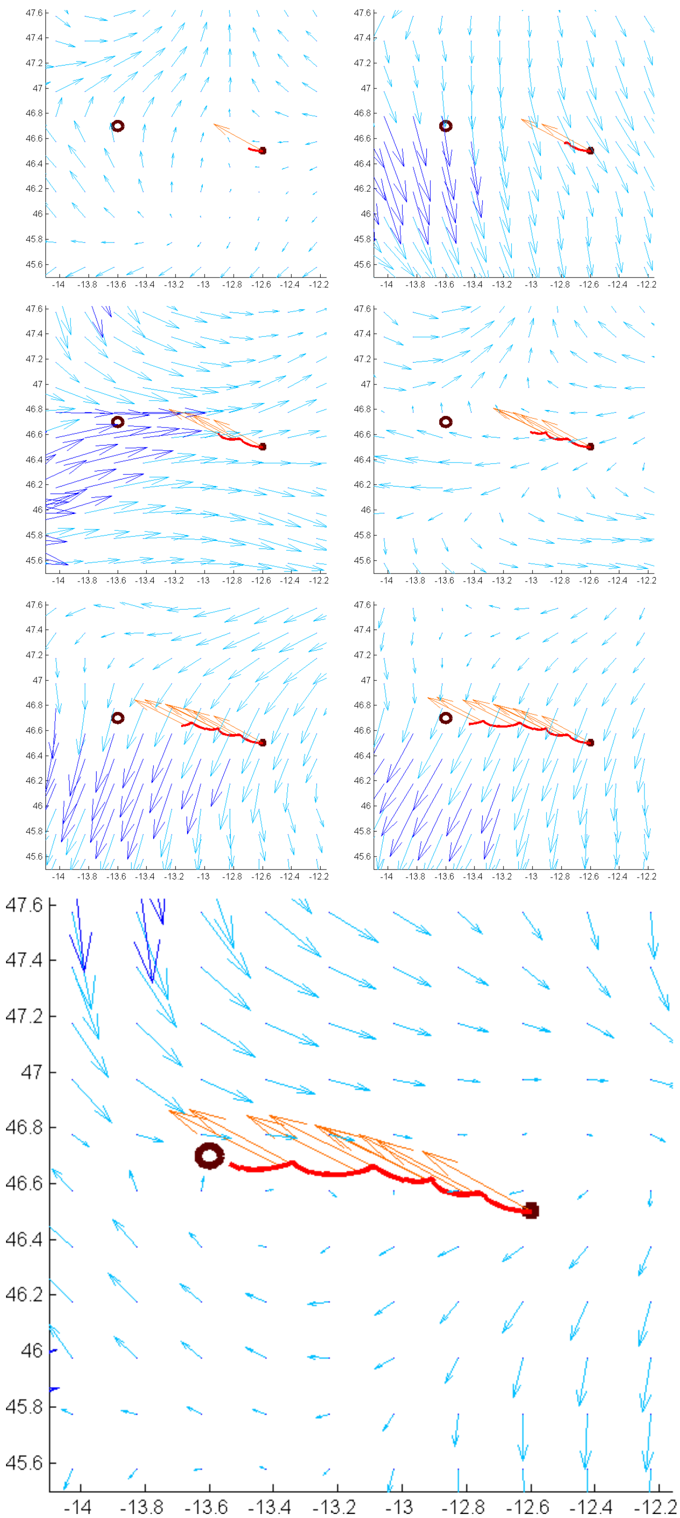

Fig. 3

SEVEN SNAPSHOTS OF THE OPTIMAL TRAJECTORY (RED LINE) SIMULATION FOR A 3-DAY TIME-BASED PROBLEM WITH TIME-DEPENDENT CURRENTS. SQUARE: START POINT. CIRCLE: GOAL POINT. LIGHT BLUE ARROWS: OCEAN CURRENTS FIELD. BLUE ARROWS: OCEAN CURRENTS THAT EXCEED THE GLIDER SPEED. ORANGE ARROWS: GLIDER BEARINGS FOR EACH SURFACING.

Algorithm 1 Path Planner

Require: The start p_1 and target p_n points, the ocean currents map $\mathcal{M}$, the glider setup $\mathcal{G}$ and the target precision radius $d_{\max}$.

Ensure: List of bearings $\mathcal{B}$ for the path of minimal cost found.

Algorithm: **pathplanner**(p_1, p_n, $\mathcal{M}$, $\mathcal{G}$) **return** $\mathcal{B}$

1: $p = p_1$ ▷ previous surface point
2: $\mathcal{B} = \varnothing$
3: $d =$ **distance**(p, p_n)
4: **while** $d > d_{\max}$ **do**
5: $\quad\hat{n} =$ **numbearings**$(p, p_n, \mathcal{M}, \mathcal{G})$ ▷ # bears. to goal
6: $\quad\hat{\mathcal{B}} =$ **inibearings**$(p, p_n, \hat{n})$ ▷ init. new bears.
7: $\quad\mathcal{B}_0 = \langle \mathcal{B}, \hat{\mathcal{B}} \rangle$ ▷ combine bears.
8: $\quad\langle \mathcal{B}, p \rangle =$ **optimize**$(\mathcal{B}_0, \mathcal{M}, \mathcal{G})$
9: $\quad d =$ **distance**(p, p_n)
10: **end while**
11: **return** $\mathcal{B}$

the first versions of the planner, diverse multi-objective cost functions were designed and tested. The results were not satisfactory, showing poor convergence and lack of robustness; as a consequence, a different multi-stage iterative scheme was designed.

The proposed method to solve the distance-based problem is presented in Alg. 1, and operates as follows. In a first initialization step, the ocean current map is analysed on the basis of a line-of-sight start to target trajectory in order to compute a coarse optimistic underestimation of the number of bearings required to reach the target (**NUMBEARINGS** function). Then a first optimization phase takes this information to compute an optimal configuration for the bearing parameters, that are initialized as described in the time-based problem (**INIBEARINGS** function). If the target waypoint is not reached in the optimization procedure (**OPTIMIZE**) with the desired precision, the bearing set is extended (see line 7). We add an underestimation of the number of bearings required, according to the remaining distance to the target (**DISTANCE**). After that, a new optimization process is launched over the complete parametrization. Hence, we take the previous optimization results as initial values for the existing bearings and default initial values for new ones. This cycle is repeated until the precision radius is satisfied.

Compared with the multi-objective optimization, this iterative scheme yields better results for both the optimality of the generated trajectory, convergence time and robustness.

B. Coordination of Multiple Vehicles using Constraints

As an example of the flexibility and scalability of our approach, here we describe how it can model multiple vehicles and impose constraints among their trajectories. This constitutes a form of coordinated path planning, in which we consider a particular constraint, but we might have adopt others.

Basically, we have extended the problems explained thus far to multiple gliders. As shown in Fig. 4, these gliders are forced

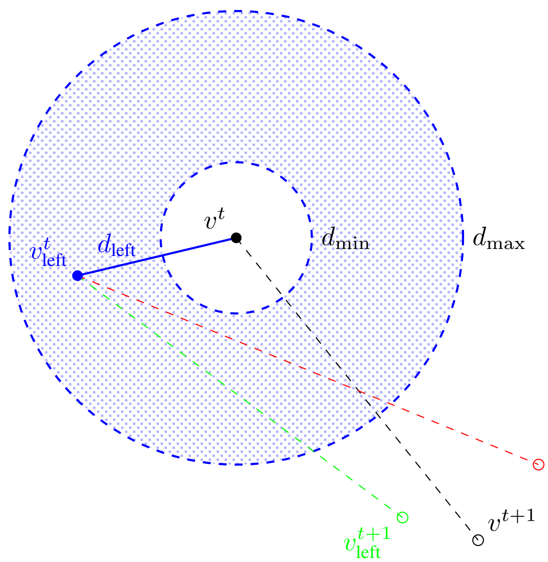

Fig. 4

EACH GLIDER SETS ITS TRAJECTORY ACCORDING TO A CONSTRAINT. IN THIS CASE, AT A GIVEN TIME t, FOR A PARTICULAR GLIDER v^t IN THE FLEET, WE IMPOSE THAT THE DISTANCE d_{LEFT} WRT THE LEFT GLIDER v^t_{LEFT} MUST LIE WITHIN THE RANGE $[d_{\min}, d_{\max}]$. ALSO, NOTE THAT FOR A GIVEN TIME t WE CHECK THAT THE STINTS DO NOT INTERSECT. FOR THIS REASON, THE STINT FROM v^t_{LEFT} TO $\hat{v}^{t+1}_{\text{LEFT}}$ IS DISCARDED. WE WILL PROCEED SIMILARLY FOR A RIGHT GLIDER v^t_{RIGHT}, IF PRESENT.

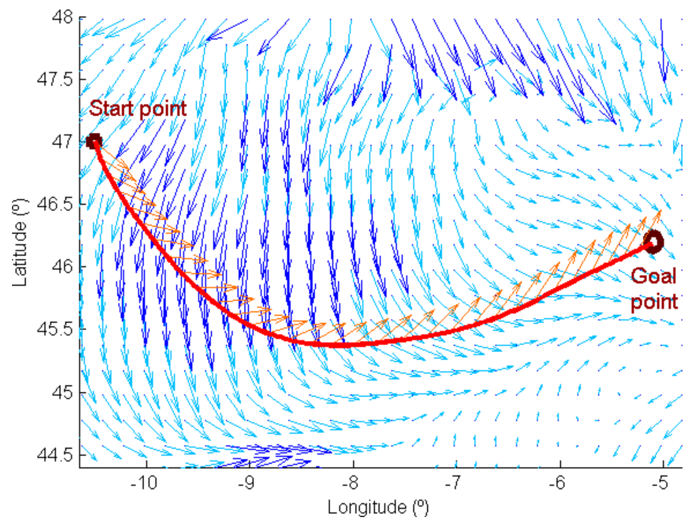

Fig. 5

EXAMPLE OF PATH PLANNING WITH OPTIMIZATION FOR A DISTANCE-BASED PROBLEM. GLIDER SPEED = 0.4m/s. DISTANCE = 423.22km. PATH TIME = 10.5 DAYS. COMPUTATIONAL COST = 24.9s. LIGHT BLUE ARROWS: OCEAN CURRENTS FIELD. BLUE ARROWS: OCEAN CURRENTS THAT EXCEED THE GLIDER SPEED. ORANGE ARROWS: GLIDER BEARINGS FOR EACH SURFACING.

to travel within a distance range among them. To be more specific, given a glider v, at each surfacing point we check that its neighbors lie in the range $[d_{\min}, d_{\max}]$. Only those paths in which that condition is satisfied are considered valid. Also, we check for segment intersection. As Fig. 4 illustrates, a test is carried out, in regards of the stints of all neighbor gliders. In the example of the figure, only the stint in which the glider v_{left} surfaces at v^{t+1}_{left} is valid then. These constraints are applied during the optimization phase.

V. EXPERIMENTAL RESULTS

The path planning method described in this paper have been evaluated with a testbench run in Matlab®. We have included a representative group of path planning algorithms in the experiments in order to validate our approach and compare its performance. These algorithms are summarized in the sequel, as well as their settings.

Regarding the testbench, this is composed of a set of simulated missions using the ocean current maps generated with the ESEOO Regional Ocean Forecasting System [27]. This Regional Ocean Model (ROM) covers the Atlantic shore of the Iberian Peninsula (ESEOAT) and the Canary Islands (ESEOCAN), which are the zones included in our testbench. It produces hourly outputs, organized in three 24h forecast bundles.

Given a starting and target waypoint, we try to solve both the time-based and distance-based problems discussed thus far, with each path planning algorithm (see Figure 3). Finally, we assume a constant glider speed of 0.2-0.4m/s and a stint duration of 8h.

For the time-based problem we have defined a temporal horizon of 3 days, resulting in the optimization of 9 bearings (for 8h stints). Fig. 3 shows an example of the results in this scenario. For the distance-based problem we use static ocean current fields, as in the example shown in Fig. 5, thus we can include RRT algorithm in the comparison. On the other hand, we use dynamic ocean currents to solve the time-based problem, to show that our method is suitable for this kind of environments.

A. Evaluated methods

Each of the methods chosen for comparison is described briefly below:

- **Direct to goal**: This is a trivial solution to the problem, i.e. without planning. As depicted in Fig. 6, in which the bearing equals the direction to the target waypoint, for each stint.
- **RRT**: The Rapidly-exploring Random Tree [10] bases on a random generation of test cases on the problem domain, building up an exploring tree with nodes that tend to cover the search space. Note that when applied to navigation, a bias is used to direct trajectories towards the target location. Our implementation is based on the work from Rao and Williams [13].
- **A***: Our adaptation to operate with ocean currents in an uniform grid is equivalent to Garau's [3] (see Fig. 7).

Also, the heuristic function is an optimistic time estimation to reach the target waypoint, given by the quotient of the distance to it and the maximum ocean current speed.

- **CTS-A***: This method is a variant of A* where the time between two consecutive surfacing intervals is ensured constant [19]. As Fig. 8 shows, it samples the bearing angles for each stint, and it accommodates both static and dynamic ocean currents.

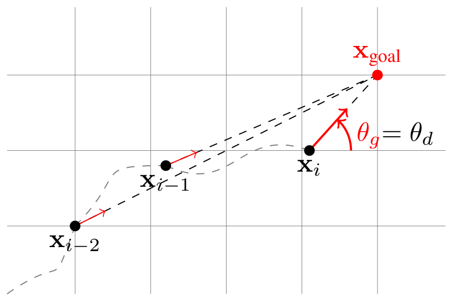

Fig. 6
DIRECT-TO-GOAL APPROACH, THAT SETS A BEARING θ_g EQUAL TO THE HEADING θ_d TO THE GOAL $\mathbf{x}_{GOAL}$ FROM THE CURRENT LOCATION $\mathbf{x}_i$

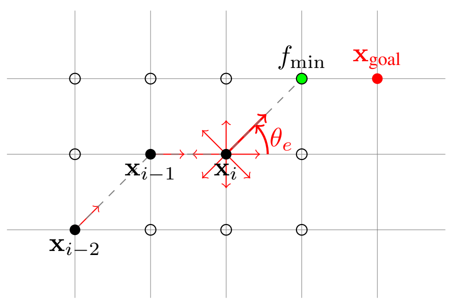

Fig. 7
CLASSICAL A* ALGORITHM OPERATING ON AN UNIFORM GRID THAT DEFINES THE SEARCH SPACE DOMAIN Ω. THE HEADING ANGLES θ_e ARE CONSTRAINED BY THE GRID, PRODUCING STRAIGHT LINE TRAJECTORIES BETWEEN NODES AND NON-CONSTANT SURFACING TIMES

B. Comparative tests

To compare the performance of each path planning method we have simulated the trajectories in the ESEOAT and ESEOCAN zone. We compute two measures to compare the methods: path quality and computational cost. The path quality is evaluated as the time required to reach the target waypoint, for each trajectory found, i.e. the travel time.

It is worth noting that A* results require special consideration, because the method used to generate the trajectories produces unrealistic non-constant stint durations, that depends on the grid resolution. That is to say, the surfacing points in A* will not generally correspond with the actual surfacing points of the glider.

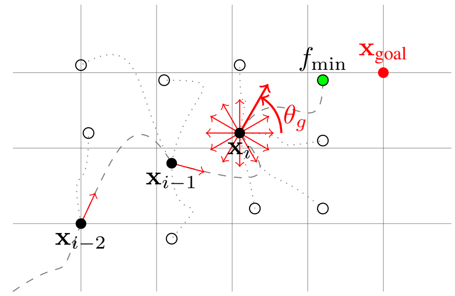

Fig. 8
CONSTANT-TIME SURFACING A* ALGORITHM. AT EACH SURFACING LOCATION $\mathbf{x}_i$ DIFFERENT BEARING ANGLES θ_g ARE CONSIDERED, INTEGRATING THE GLIDER TRAJECTORY FOR THE SURFACING TIME t_s

TABLE I
MEAN NUMBER OF DAYS TO REACH THE GOAL

Methods	Mean of all cases	Mean selected cases
Direct to goal	No arrival in 35%	18.0
RRT	No arrival in 5%	17.7
A*	18.9	17.1
CTS-A*	18.7	16.9
Optimization	18.4	16.7

The computational cost is of certain importance in glider missions, as sometimes it is mandatory to find a path in a few minutes. For instance, when an unforeseen risky situation occurs while the glider is surfacing, a new bearing must be computed before the glider initiates the next stint.

Regarding the algorithms' parametrization used in the comparison, we have used the same equivalent discretization level for each method, when applicable. In fact, the spatial grid for A* and CTS-A* has $1/20$ degrees of resolution. And CTS-A* uses a division of $20°$ in the bearing rose.

1) Distance-based Scenario: We have compared the methods optimizing the path between two waypoints under the same conditions for all methods, i.e. the same glider configuration and ocean current field. A set of 20 cases has been analysed.

Table I summarizes the path cost in terms of the mean days required to reach the target waypoint for each method. In some test cases, several algorithms did not find a solution, thus we have included a third column to show the mean travel time of those cases in which all the methods found a solution.

Table II shows the computation time for each method, measured on a Intel® Core™ 2 Quad processor computer running at 2.5GHz. Figures 9 and 10 present examples of short and long paths obtained by each method for two of the cases analysed.

These experimental results show that the path planner proposed in this paper obtains the best results, in terms of path optimality and computational cost. The paths generated using this method are between 2% and 5% faster than the ones produced by others. Additionally, methods like Direct to

TABLE II

MEAN COMPUTATIONAL COST (MINUTES)

Direct to goal	< 1
RRT	< 1
A*	9.5
CTS-A*	77.1
Optimization	3.5

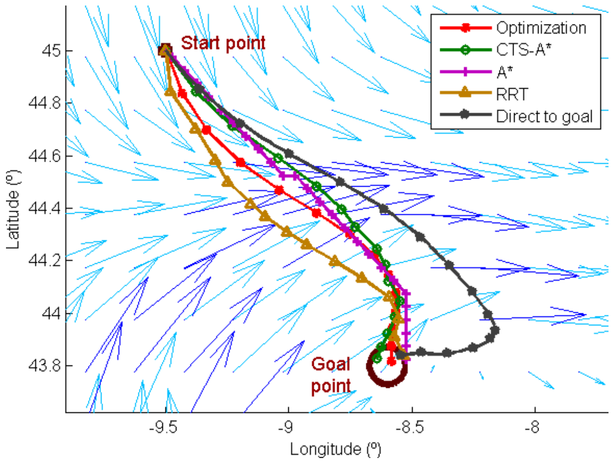

Fig. 9

COMPARATIVE OF TRAJECTORIES FOR A DISTANCE-BASED PROBLEM. GLIDER SPEED = 0.4m/s. DISTANCE = 151.4km. LIGHT BLUE ARROWS: OCEAN CURRENTS FIELD. BLUE ARROWS: OCEAN CURRENTS THAT EXCEED THE GLIDER SPEED. TRAVEL TIME (DAYS): OPTIM.: 4.0; CTS-A*: 4.3; A*: 4.5; RRT: 4.6; DTG: 4.9.

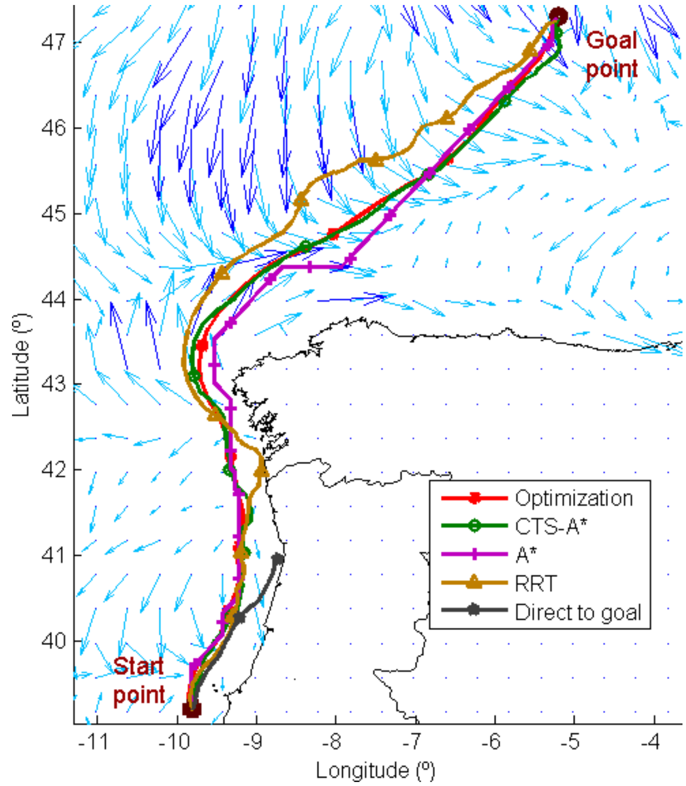

Fig. 10

COMPARATIVE OF TRAJECTORIES FOR A DISTANCE-BASED PROBLEM. GLIDER SPEED = 0.4m/s. DISTANCE = 974km. TRAVEL TIME (DAYS): OPTIM.: 26.3; CTS-A*: 27.2; A*: 27.3; RRT: 31.3; DTG: NO ARRIVAL.

Goal and RRT are not successful in solving all the cases tested. Regarding computational cost, our approach runs 3 times faster than A* method and 20 times faster than CTS-A*.

If we used a smaller grid size in the A* method or if we increased the number of bearings that can be explored in CTS-A* method, the results of these would improve slightly; however, the computational cost would increase notably. On the contrary, using our algorithm a close to optimal path can be found in a reasonably low computational time.

Finally, in Fig. 11 we include an example of the trajectories that are computed in adverse conditions, i.e. for a glider nominal speed has been reduced to 0.2m/s. Recall that a glider with low speed drifts more in the direction of ocean currents.

2) Time-based Scenario: To compare the performance of each path planning method in this scenario, we have optimized the paths towards a distant waypoint using 3-day forecast horizon. We use the remaining distance from the final glider position to the target waypoint to measure the results (the lower the better). A total of 65 cases have been analysed.

The RRT algorithm is not suitable for this scenario, and therefore is not included in the analysis. The RRT search can be expanded only from the starting point on a time-dependent ocean current field. In the case of A* algorithm, the explored vertices of the search graph can no longer be discarded as a consequence of the temporal variation of ocean currents. Assuming this limitation, we have run A* in this scenario though.

In Fig. 12 and 13 illustrative examples are presented. In the figures, the last ocean current field is shown. In a video sequence it is possible to interpret the correlation between the trajectory and ocean currents.

TABLE III

MEAN DISTANCE DIFFERENCE (km) WRT DIRECT TO GOAL TO REACH THE GOAL

Methods	All	Strong	Weak
A*	0.5	4.9	-6
CTS-A*	5.2	8.6	0.2
Optimization	8.5	12.4	2.7

Table III shows the mean distance differences with respect to Direct to Goal, for the whole set of test cases. In all of them, the proposed optimization-based method gets better results, being able to generate paths that end closer to the target. We have also split the cases in two groups, according to the strength and direction of ocean currents. Strong cases are those

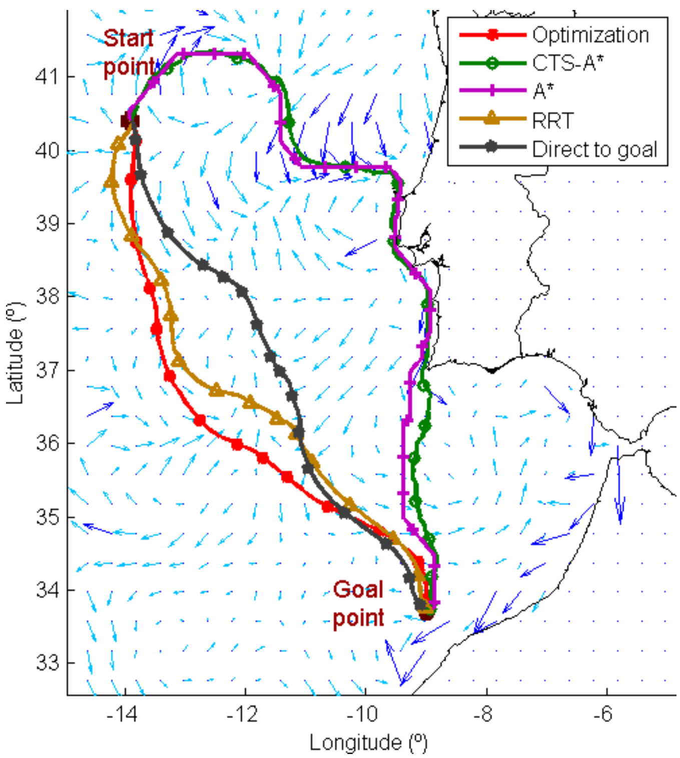

Fig. 11
COMPARATIVE OF TRAJECTORIES FOR A DISTANCE-BASED PROBLEM. GLIDER SPEED = 0.2m/s. DISTANCE = 861.88km. LIGHT BLUE ARROWS: OCEAN CURRENTS FIELD. BLUE ARROWS: OCEAN CURRENTS THAT EXCEED THE GLIDER SPEED. NUMBER OF DAYS OF PATHS: OPTIM.: 47.4; CTS-A*: 50.0; A*: 49.6; RRT: 49.7; DTG: 53.8.

TABLE IV
MEAN COMPUTATIONAL COST (SECONDS)

Direct to goal	< 0.1
A*	38.4
CTS-A*	342.4
Optimization	8.1

cases in which ocean currents are against the direction to the target waypoint and faster than the glider speed v_g, while weak cases are the rest —i.e. slower than v_g or aligned with the direction to the target waypoint. As expected, the improvement in the paths found is always greater in strong cases, for all methods. Note that A* results must be interpreted carefully. The discretization of the search space and, especially, of the bearing angles —in multiples of $45°$— justifies that in certain cases the optimum is not found, since it lies in the continuous space. Although it is possible to increase the spatial resolution and consider a neighborhood radius $r > 1$, to alleviate this problem, the computation cost would explode. In fact, it is already substantially higher than the optimization method, as Table IV shows.

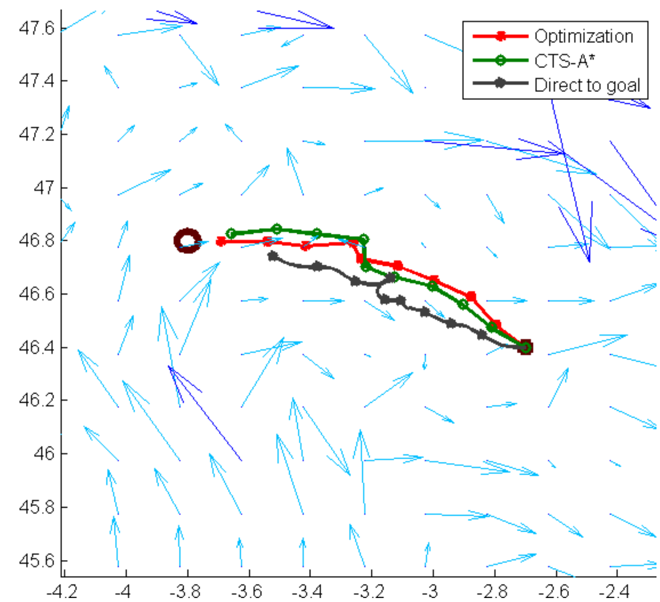

Fig. 12
COMPARATIVE OF TRAJECTORIES FOR A TIME-BASED PROBLEM. GLIDER SPEED = 0.4m/s. DISTANCE = 95.3km. DISTANCE TO REACH THE TARGET: OPTIM.: 8.4km; CTS-A*: 9.9km; DTG: 22.5km.

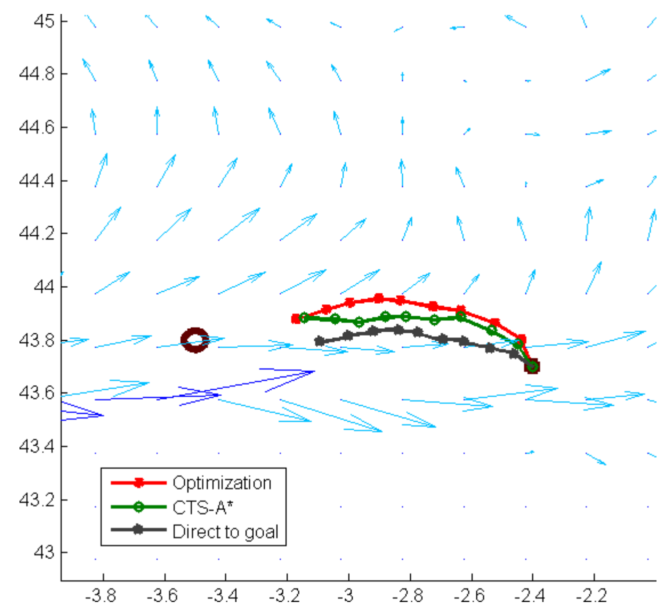

Fig. 13
COMPARATIVE OF TRAJECTORIES FOR A TIME-BASED PROBLEM. GLIDER SPEED = 0.4m/s. DISTANCE = 89.3km. DISTANCE TO REACH THE TARGET: OPTIM.: 27.7km; CTS-A*: 29.6km; DTG: 32.8km.

C. Multiple Glider Coordination

Regarding the coordinated path planning problem discussed in Sec. IV-B, here we show a particular case for a fleet of 3 gliders (see Fig. 14 and 15). In the optimization process, the distance range ($[5, 15]\mathrm{km}$ in the figure) is checked for each surfacing point and also intersections among trajectories are forbidden. In brief, we optimize the gliders' trajectories subject to such constraint. The figures illustrate that the paths not just satisfy the constraint, but also minimize the travel time.

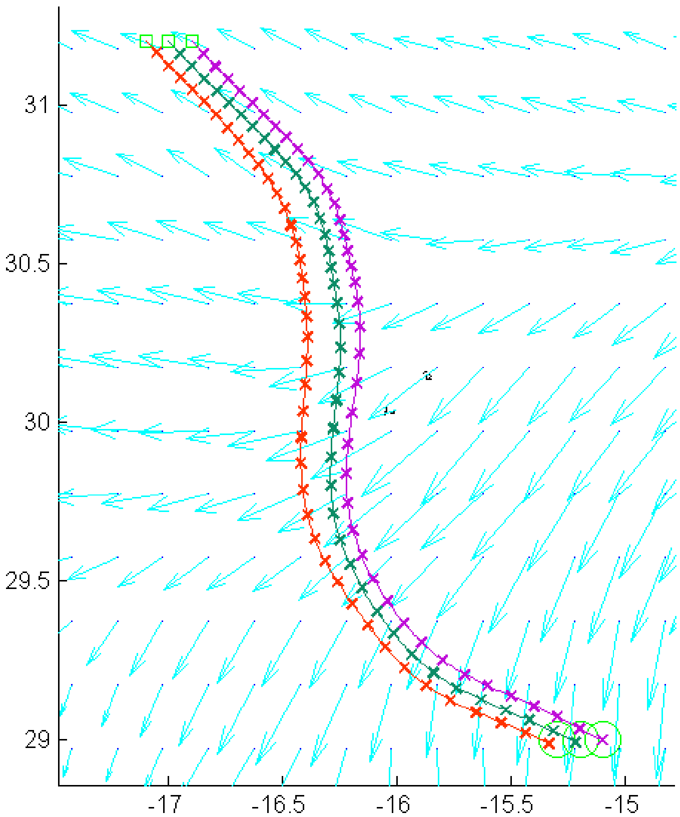

Fig. 14

PATHS FOUND FOR A FLEET OF 3 GLIDERS THAT MUST TRAVEL WITHIN A DISTANCE RANGE WRT THE NEIGHBOR GLIDERS. GAP OF 0.1° AMONG START □ AND TARGET ○ WAYPOINTS. DISTANCE = $299\mathrm{km}$.

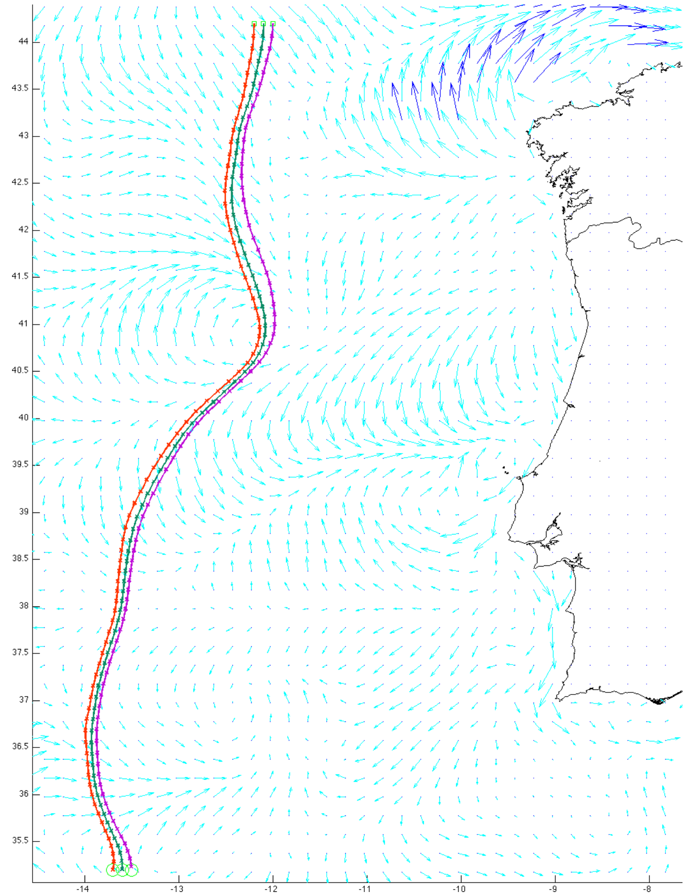

Fig. 15

PATHS FOUND FOR A FLEET OF 3 GLIDERS TRAVELLING WITHIN THE DISTANCE RANGE $[5, 15]\mathrm{km}$ WRT THE NEIGHBOR GLIDERS. GAP OF 0.1° AMONG START □ AND TARGET ○ WAYPOINTS. DISTANCE = $1008\mathrm{km}$.

VI. CONCLUSIONS

We have described a novel path planning algorithm for gliders based on optimization that offers promising results on realistic simulations. Contrary to other approaches, our proposal models the vehicle navigation faithfully, assuming a bi-dimensional realm. The experiments show a superior performance when compared with other path planning methods.

In general, classical A* or variants, like the CTS-A* algorithm analysed here, and sampling-based methods like RRT do not find paths better than our iterative optimization methods in a reasonable processing time. The reduction in the travel time might diminish the economical cost of the mission. Anyhow, it is in the computational cost where the latter clearly outperforms the former, permitting the planner execution in the reduced interval the glider surfaces before a new stint is started. Or, for example, path planning for different departure times, in order to postpone the deployment to the most favourable moment.

Our solution can solve two different path planning problems. The time-based one, that tries to left the vehicle at the closest possible distance towards a target waypoint in a given time, and therefore where the number of bearings required is known. And the distance-based one, that pursues to reach the target waypoint in the minimum time, hence the number of bearings required is unknown in advance.

We have shown that our approach can be applied to solve those problems with multiple gliders, whose trajectories might be constrained. In this work we have presented a solution for a common situation in which a fleet of gliders must cover a route in parallel. In such situation, the gliders must maintain a minimum distance between them to avoid collisions and a maximum distance between them to perform a dense sampling across the route.

The experimental results show that our proposal gives better results than the path planning algorithms included in the comparison. We have obtained more optimal path in less computation time, both for static and time-dependent ocean currents. Additionally, this method allows for path re-planning in real conditions, as it can be executed in the few minutes a glider is on surface between stints. Also, the method is quite flexible, as it can be applied to a number of other optimization problems with few adaptation or configuration.

VII. Further Work

Further work will consist in improving the initialization step of our iterative optimization-based path planner, in order to avoid particular situations in which it might get trapped in local minima. This is the case of coastal ocean environments, where some kind of obstacle avoidance must be applied in the optimization process. Also, the extension to three-dimensional environments, with an adequate glider motion model, is of interest to analyse the scalability of our approach.

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