Wind blade chord and twist angle optimization by using genetic algorithms

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Keywords: Optimal Design, Evolutionary Algorithms, Energy Systems, Wind Power, Computational Fluid Dynamics.

This paper shows a method to obtain optimal chord and twist distributions in wind turbine blades by using genetic algorithms. The distributions are computed to maximize the mean expected power depending on the Weibull wind distribution at a specific site because in wind power systems optimization is highly site dependent [1]. This approach avoids assumptions about optimal attack angle related to the ratio between the lift to drag coefficients.

Genetic algorithms are global optimizers that have a wide trade-off between exploration and explotation on the space problem. The geometry definition of a wind blade is a problem with many degrees of freedom being suitable to fall in local optima, which can be surpassed using evolutionary methods. Evolutionary Algorithms are frequently used as powerful optimization methods. They are stochastic methods inspired in the natural process of evolution[2, 3], and among their advantages are their global search due to the management of a population of candidate solutions instead only one, and also the only requirement of the knowledge of the fitness function value to perform a evolutionary optimization, without any other consideration such as derivability or continuity of the function. Many different optimum design problems in multiple fields of sciences and engineering have been solved outperforming any other previous results with evolutionary algorithms [4].

To optimize chord and twist distributions, an efficient implementation of the Blade-Element and Momentum(BEM) theory [1, 5, 6] is used. It is basically a simplified theory that is used routinely by wind power industry because it provides reasonably accurate prediction of performance. The BEM theory has shown to give good accuracy with respect to time cost, and at moderate wind speeds, it has sufficed for blade geometry optimization.

In the implementation of BEM, the sophistication is dismiss to reduce computational cost. The time required to evaluate the forces in a typical turbine is in the order of milliseconds, which allows massive evaluation of trial turbines. The implementation is validated by comparing power prediction with the experimental data of the

Risø test turbine that is one of six experimental turbines widely tested by the IEA. The data are contained in the Annex XVIII report [7] and in the public database of rotor performances at the ECN.

High quality in results is obtained until the stall zone, about wind speed of 13m/s proximately. Predictions are used to compute the mean power that is the fitness function in the genetic algorithm. The mean power, which is proportional to the annual generated energy, is obtained by averaging power predictions with the probability obtained from the Weibull distribution of the specific site. To obtain the optimal blade, the upper and lower limits of chord and twist are needed as well as an optional upper limit of the blade area.

An application is presented to optimize the blades of the Risø test turbine at a site with wind distribution parameters: $A = 8.6$ and $k = 2.66$. Optimized blades have more torsion and the collective pitch angle is changed from 1.8^o to -2.986^o. A slight redistribution of chord is generated with decreasing values in both external sections. Results show that optimized turbine has better performance for wind speed until 14.5m/s and worse for higher values, but these are less probable.

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Abstract

This paper shows a method to compute the chord and twist distributions in wind power blades. The distributions are computed to maximize the mean expected power depending on the Weibull wind distribution at a specific site. This approach avoids assumptions about optimal attack angle related to the ratio between the lift to drag coefficients. To optimize chord and twist distributions, an efficient implementation of the Blade-Element and Momentum theory is used. In the implementation, the sophistication is dismiss to reduce computational cost. The time required to evaluate the forces in a typical turbine is in the order of milliseconds, which allows massive evaluation of trial turbines. The implementation is validated by comparing power prediction with the experimental data of the Risø test turbine. High quality in results is obtained until the stall zone, about wind speed of 13m/s proximately. Predictions are used to compute the mean power that is used as the fitness function in a genetic algorithm. An application is presented to optimize the blade of this test turbine for a specified wind distribution.

Keywords: Optimal Design, Evolutionary Algorithms, Energy Systems, Wind Power, Computational Fluid Dynamics.

1 Introduction

The power efficiency of wind energy systems has a high impact in the economic analysis of this kind of renewable energies. The efficiency in these systems depends on many subsystems: blades, gearbox, electric generator and control. Some factors involved in blade efficiency are the wind features, eg. its probabilistic distribution, the mechanical interaction of blade with the electric generator, and the strategies dealing with pitch and rotational speed control. It is a complex problem involving many factors, relations and constraints.

The design of optimal blades involves aerodynamical, structural and control problems. However, the design cycle can be practically approached as an iterative and stepped method. For aerodynamical optimization the blade can be modeled as a serie of sections along the pitch axis. Each section has an airfoil shape, chord length and attach angle which is the result of a collective pitch angle and a local twist one. This last is a property of the blade while the pitch angle depends on the control strategy of the whole energy system.

Airfoil shape optimization have been widely studied with many approaches including genetic algorithms. Most of them use finite shape representation which is extended by splines and attempt to maximize the lift to drag ratio. Blade optimization comprises also chord and twist determination along the blade axis. Practical values for both of them have been obtained by supposing constant attack angle along all the blade, that means, constant lift and drag coefficients. However, the attack angle cannot be considered constant in the different aerodynamic environments ranging from the nearest to further sections of rotor axis, as well as in different wind speed of the Weibull distribution. It is posible to obtain an optimal blade for a selected aerodynamic environment, but the optimal blade for a long test time period, as several years, requires the computation of the statistical expectation values obtained from the simulation in several aerodynamic environments.

This paper includes a procedure to solve the chord-twist optimization by using genetic algorithms, which implies the simulation on a wide range of aerodynamic environments. The user has to define the airfoil shapes, the parameters of the Weibull distribution, and some mechanical properties as the minimum and maximum values of chord and twist angle. In practice, rather than defining the airfoil shape itself, the user has to include tables for lift and drag coefficients depending on the attack angle. Both of them must cover the aerodynamic range considered in the simulation.

The blade element theory is used to determine the torque and power. The coupling between the wind flow and the rotating blade requires the use of the axial and tangential induction factors for each section along the blade axis. They are obtained by solving a nonlinear equation system. The power is obtained for each wind speed according to two defined strategies for pitch and rotational speed control. These strategies are fixed values, stall regulation, or optimal ones. The mean power computed from the wind speed probabilistic distribution is used as the fitness function for the genetic algorithm.

The computation of the wind flow around rotating blades is a very complex problem. For a precise knowledge of the wind flow and the induced forces in the turbine surfaces it is necessary to solve the three-dimensional Navier-Stokes equations in a rotating frame, but the computational cost to obtain such precise solution prohibits their use in the design and analysis environments [1]. Moreover, any procedure used to obtain optimal designs must be very efficient in time cost because the optimization cycle requires many trials.

For wind power applications, we are interested in global momentum changes that generate thrust and torque in the rotor, rather than an exact knowledge of the flow. The blade element momentum theory(BEM) [1, 2, 3] is basically an one-dimensional simplified theory that is used routinely by wind power industry because it provides reasonably accurate prediction of performance. The BEM theory has shown to give good accuracy with respect to time cost [4], and at moderate wind speeds, it has sufficed for blade geometry optimization [5].

The BEM theory is the composition of two different approaches to study the forces in a wind turbine. The first is the momentum theory that studies the global changes in wind momenta, axial and tangential, in an ideal turbine. Changes in axial and rotational momenta between upwind and downwind induce thrust and torque respectively in the rotor. The wind flow is split in many differential noninteracting annular stream tubes. Therefore, the complexity of the aerodynamic analysis is reduced to one dimensional problem by analyzing only a disc of differential dr wide at radius r from the rotation axis. As the flow, the turbine blades are divided into a number of independent infinitesimal elements, of wide dr, along the length of the blade.

The second theory, the blade element, studies the aerodynamic forces acting in a local airfoil. As in aeronautics wing theory, the forces are lift, which is perpendicular to the wind direction, and drag that is in the same direction. Drag is mainly generated by friction between the viscous fluid and the airfoil surface. It is a dissipative force that generates power loss and lack in momentum changes.

Different BEM codes have been implemented and tested, but the differences among their results and that provides by other methods results are mainly generated by the differences among airfoil data[6]. Although there are some uncertainty on the results of BEM method, it is commonly used in blade design[7]. Today there is not a definitive and precise computational method for aerodynamic evaluation of wind turbines[7, 8], but BEM is the most efficient in time cost.

Genetic algorithms are global optimizers that have a wide trade-off between exploration and explotation on the space problem. The geometry definition of a wind blade is a problem with many degrees of freedom being suitable to fall in local optima, which can be surpassed using evolutionary methods. Evolutionary Algorithms are frequently used as powerful optimization methods. They are stochastic methods inspired in the natural process of evolution[9], and among their advantages are their global search due to the management of a population of candidate solutions instead only one, and also the only requirement of the knowledge of the fitness function value to perform a evolutionary optimization, without any other consideration such as derivability or continuity of the function. Many different optimum design problems in multiple fields of sciences and engineering have been solved outperforming any other previous results with evolutionary algorithms, both in single objective[10, 11, 12] and multiple objective optimization[13, 14].

Blade optimization has been carried out from different approaches. Fuglsand and Madsen[15] use a global methodology involving many aspects of blade design including aerodynamics, blade structure, fatigue loads, noise generation and economical costs that are used to define an objective function as the ratio of the total cost of the turbine to the annual energy production. This objective function is optimized by using gradient based methods. Benini and Toffolo[16] use evolutionary methods to optimize wind turbines. They fix the turbine power and include in the objective function economical costs.

A very complete approach to blade optimization is the realized by Hampsey[17]. His optimization procedure includes changes in the airfoil shape, but to avoid the computational cost involved in obtaining lift and drag tables, he uses a 3D panel method to obtain the force distributions. The CFD procedures are excluded because their excessive computational cost and XFOIL[18] is also excluded because has high computational cost and mainly has many uncertainty with its convergence problems. However, this approach dismisses the role of realistic drag data in the turbine evaluation.

The approach of this paper is less ambitious that other proposals. It is based in three simplificative decisions. The first is to concentrate only in the aerodynamic problem, because the optimal design should be obtained after many refinement cycles including each one many multidisciplinary aspects as aerodynamics, structural, control and economical successive steps. The second decision is to use a non sophisticate prediction method as a simple implementation of BEM, which is simple but high computational efficient. The third decision is to avoid the shape modification that overload the computational cost and that get, into the turbine evaluation, uncertain and imprecise data. This decision allows the use of high quality data for lift and drag obtained from experimental tests.

There are some available BEM codes, eg. WT_Perf, but they are independent programs that use file interchange for input and output interfaces. Therefore, most of their efficiency is lost in that type of data interchange. Optimization with evolutionary methods requires many trial evaluations. Therefore, we have implemented a BEM procedure for a better and direct interface between genetic and BEM codes. The most interesting approach, and also the hardest one, will be to use software engineering into the Fortan sources of these available codes to generate high efficient interfaces, eg. at object modules or library level.

The Hansen[2] presentation is used as guideline in the implementation of BEM, but including some variations. A validation procedure of BEM code is included in the papper by using the Risø test turbine[19] to contrast theoretical vs experimental results. Finally, an application of the method is presented. It tries to optimize the chord and twist of this test turbine for a defined site characterized by its Weibull wind distribution parameters.

2 Blade Element Momentum Theory

The basis of BEM theory is well established, but there are some differences among final BEM models because several strategies are used to solve the non linear equations involved in this methodology, and also because many corrections are proposed to increase the precision of predictions.

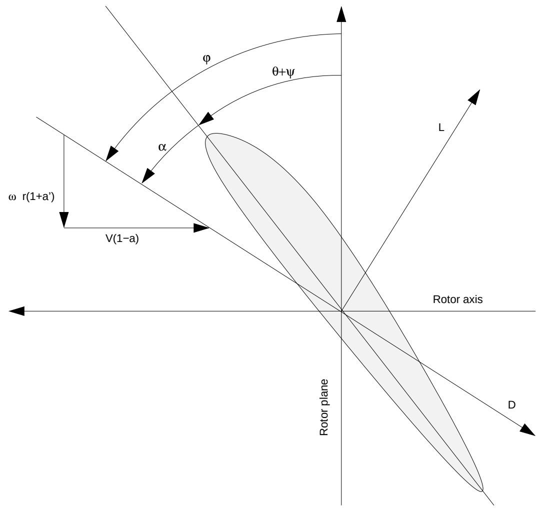

Figure 1: Blade geometry of a section at r. Angles are related to the rotor plane. The chord line orientation, $\theta + \psi(r)$, is the result of the collective pitch angle θ and the local twist angle $\psi(r)$. The angle ϕ between the relative wind in the airfoil and the rotor plane is a function of both the effective axial speed $V(1-a)$ and the effective tangential speed $\omega r(1+a')$ where a and a' are the induction factors between the turbine and wind flow.

The geometry of wind airfoil interaction is shown in Figure 1. The upstream wind speed is V, but the effective axial wind speed at the blade is $V(1-a)$ where a is the axial induction factor. The blade is rotating at the rotational speed ω, but the effective rotation between the blade and wind flow is $\omega(1+a')$, where a' is the tangential induction factor. Both induction factor are introduced to model the wind and the rotating turbine interaction. The axial induction coefficient a is due to the downstream tube expansion, while that a' the rotational or tangential induction factor is due to the induced wake rotation in the opposed direction to the rotor. The effective wind speed and orientation relative to the airfoil at r section are:

$$v = \sqrt{V^2(1-a)^2 + \omega^2 r^2 (1+a')^2} \quad (1)$$

$$\tan\phi = \frac{V(1-a)}{\omega r(1+a')} \quad (2)$$

By introducing the local tip ratio λ_r at r, the relative speed is:

$$v = V\sqrt{(1-a)^2 + \lambda_r^2(1+a')^2} = V(1-a)/\sin\phi \qquad \lambda_r = \frac{\omega r}{V} \quad (3)$$

The attack angle α between the relative wind direction and the blade is: $\alpha = \phi - \theta - \psi(r)$, where θ is a collective pitch angle, which is a parameter used for speed control, and $\psi(r)$ is the twist angle at section r. In stall-regulated wind turbines,

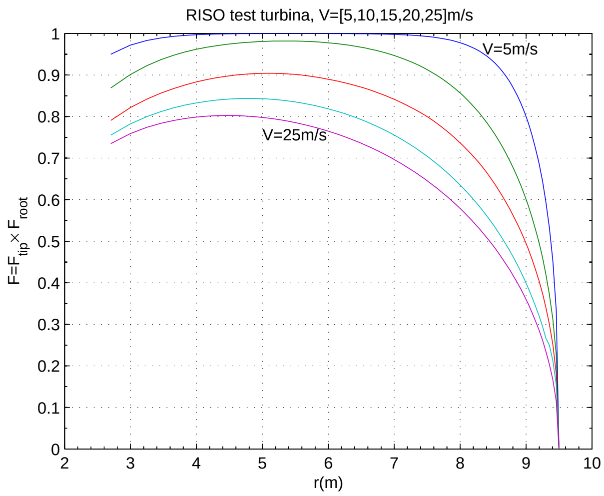

Figure 2: Correction factor in an experimental turbine for different wind speed obtained from combined effects of both tip and root corrections

without pitch control, the θ parameter is a constant. The torque in rotor axis generated by a differential section dr is dM:

$$dM = F\rho V\omega r^2 4a'(1-a)\,\pi r dr \tag{4}$$

where F is a correction factor expressed as: $F = F_{tip} \times F_{root}$. The first term is the Prandtl tip loss correction factor and the second is the root correction factor:

$$F_{tip} = \frac{2}{\pi}\arccos\left(e^{-\frac{B(R-r)}{2r\sin\phi}}\right) \qquad F_{root} = \frac{2}{\pi}\arccos\left(e^{-\frac{B(r-R_{root})}{2r\sin\phi}}\right) \tag{5}$$

where B is the blade number and R the tip radius. The momentum theory is an ideal model based in an infinite number of differential blades, but in practical turbines the blade number has a low finite value, two or three in most cases. The Prandtl factor is a correction factor for both the finite blade number and the finite blade radius. At the end of the blade, $r = R$, the wind flows from the high pressure side to the low pressure one generating an undesirable vortex with an aerodynamic loss, and at the end of blade is verified: $F(R) = 0$. A similar reason is valid for the blade root at R_{root} radius, but this correction factor is less significative. The Figure 2 shows the correction factor for several wind speed in a experimental turbine. The total torque in the turbine is produced by a tangential force F_t:

$$dM = F_t\, r dr \qquad F_t = 4\pi r^2 F\rho V\omega a'(1-a) \tag{6}$$

The term $rF_t(r)$ is the torque density. Also, the wind turbine has a thrust dT:

$$dT = F\rho V^2 4a(1-a)\,\pi r dr \tag{7}$$

The blade element theory studies a local airfoil similarly as used in wing theory for aeronautic applications. Based on the Reynolds number and attack angle we can obtain the aerodynamic forces of an airfoil. Reynolds number is defined as:

$$Re = \frac{1}{\nu} vc(r) \tag{8}$$

where ν is the cinematics viscosity, v is the relative wind speed and $c(r)$ is the chord at the section. With the attack angle α we can obtain the lift and drag coefficients: $C_L(Re, \alpha)$ and $C_D(Re, \alpha)$ respectively. They can be obtained from empirical data, wind tunnel simulation or computer generated tables. In practice both lift and drag dependence from Reynolds number is lower than compared with the attack angle. This allows to use only attack angle dependence $C_L(\alpha)$ and $C_D(\alpha)$. The torque and thrust generated in the airfoil are:

$$dM = \frac{1}{2}\rho v^2 B \left[C_L \sin\phi - C_D \cos\phi\right] c(r) r dr = \frac{1}{2}\rho v^2 B C_t c(r) r dr \tag{9}$$

$$dT = \frac{1}{2}\rho v^2 B \left[C_L \cos\phi + C_D \sin\phi\right] c(r) dr = \frac{1}{2}\rho v^2 B C_n c(r) dr \tag{10}$$

where normal and tangential coefficient are used based on the lift and drag ones:

$$C_n = C_L \cos\phi + C_D \sin\phi \qquad C_t = C_L \sin\phi - C_D \cos\phi \tag{11}$$

The BEM method is based on the correctness of both theories, momentum and blade element. Therefore, for the torque the Equation (4) has to be equal to the Equation (9), and also for the thrust the Equation (7) has to be equal to Equation (10). It is obtained that:

$$\frac{a}{1-a} \sin^2\phi = \frac{\sigma}{4F} C_n \tag{12}$$

$$\frac{a'}{1-a} \sin^2\phi \lambda_r = \frac{\sigma}{4F} C_t \tag{13}$$

where σ is the local solidity

$$\sigma = \frac{Bc(r)}{2\pi r} \tag{14}$$

This factor σ is the fraction of the tube radius length $2\pi r$ occupied by the turbine blades $Bc(r)$. Both induction factors, axial and tangential, are obtained from Equation (12) and (13) as:

$$a = \frac{\sigma C_n}{4F \sin^2\phi + \sigma C_n} = \frac{1}{\frac{4F \sin^2\phi}{\sigma C_n} + 1} \tag{15}$$

$$a' = \frac{\sigma C_t}{4F\sin\phi\cos\phi - \sigma C_t} = \frac{1}{\frac{4F\sin\phi\cos\phi}{\sigma C_t} - 1} \tag{16}$$

Some authors[1, 3] have proposed that the drag coefficient should not be included in the induction computation. This means that we can choose the inclusion or exclusion of the drag in the previous equations, equivalent to change: $C_D \rightarrow \kappa C_D$, where $\kappa \in \{0, 1\}$. An additional refinement is used, which is based on a better modeling of empirical results. If K is the nondimensional number that appears in the axial induction factor expression:

$$K = \frac{4F\sin^2\phi}{\sigma C_N} \tag{17}$$

The Glauert's correction for the Equation (15) is:

$$a = \begin{cases} \frac{1}{K+1} & a \leq a_c \\ \frac{1}{2}\left[2 + K(1-2a_c) - \sqrt{(K(1-2a_c)+2)^2 + 4(Ka_c^2 - 1)}\right] & a > a_c \end{cases} \tag{18}$$

where $a_c = 0.2$ is a upper bound of validity of the basic BEM theory. Right side of Equations (18) and (16) are functions of both a and a', therefore, a non linear system is obtained:

$$a = H_n(a, a', r, \psi(r), c(r), \omega, V, \theta, B, R, R_{root}) \tag{19}$$

$$a' = H_t(a, a', r, \psi(r), c(r), \omega, V, \theta, B, R, R_{root}) \tag{20}$$

For every section at r, there is a nonlinear equation system of two equations with two unknown variables:

$$a - H_n(a, a', \ldots) = 0 \qquad a' - H_t(a, a', \ldots) = 0 \tag{21}$$

To solve this problem two approaches are possible. The first is based on computing the serie of successive values where $n+1$ result are based on n as: $a^{(n+1} = H_n(a^{(n}, a'^{(n}, \ldots)$ until the reaching of some convergence criterium. The second is based on the zero finding problem. The Newton-Raphson procedure is usually used to solve one dimensional problem of zero finding, but for many dimensional ones pseudo-Newton method is used. The Matlab `fsolve` procedure can be used to solve this system. Internally it tries to minimice the following function:

$$\min_{a,a'} Z(a, a') = [a - H_n(a, a', \ldots)]^2 + [a' - H_t(a, a', \ldots)]^2 \tag{22}$$

In the two approaches, the procedure stops when a precision level is achieved for both induction factors: $a^{(n+1} - a^{(n} < \epsilon$ and $a'^{(n+1} - a'^{(n} < \epsilon$ or when the maximum iteration number is used without reaching the desired convergence. The implementation of BEM code has no convergency in some sections. In such case, the value of $F_t(r)$ corresponding to station at r is obtained by means of cubic interpolation from its neighbors stations. If the number of nonconvergent stations is greater than the 20% of stations, this is a global nonconvergent turbine configuration. In that case, this infeasible solution candidate is deleted and ignored by the evolutionary algorithmic.

By integrating dM along the radius we can obtain the total turbine torque M. The Equation (6) is used with a linear law for F_t among different sections[2]. The torque $M(\mathbf{g}, V, \omega, \theta)$ is a function of all the geometric factors, which we express as g, and the global variables V, ω and θ. The turbine power is computed as:

$$P(\mathbf{g}, V, \omega, \theta) = \omega M(\mathbf{g}, V, \omega, \theta) \tag{23}$$

2.1 Validation and improvement of BEM implementation

The validation of the BEM implementation requires to contrast the prediction for the performance of a real turbine with its experimental results. Both the turbine and the experimental data must to be unambiguous and precisely defined to avoid interpretative errors. To validate the code, we use the available data of the Risø test turbine that is one of six experimental turbines widely tested by the IEA and included in the Annex XVIII report[19]. Data are available for the research community in a public database in the ECN. The test turbine has a 19m rotor diameter with 100Kw of nominal power. The turbine includes three LM 8.2 blades that are chord, thickness and twist variable. The blade airfoils are in the NACA 63-2XX family. Precise data for the lift and drag are provided for the thickness numbers $\{25, 21, 18, 15, 12\}$. The active part of the blade starts at the 2.7 m section and finishes at 9.5 m. The maximum twist is 15^o at first active section and null at the last. The chord value decreases linearly along the blade radius. The thickness, chord and twist data are contained in the Table 2.

This test turbine is stall-regulated. Its pitch is fixed and it rotates at constant angular speed ω. The asynchronous generator has a double wound that allows a double rotational speed of 35.6 rpm or 47.5 rpm. Database in the ECN contains many files with experimental results suitable for comparing with predictions. The database entry '/data/risoe/rotcoef/coef_r.zip' includes the file 'PV_r' with the mechanical and electric power of the turbine for different wind speeds. For testing computational vs experimental data, the mechanical power is used because the electric one is depending on the generator efficiency.

No good predictions are obtained by computing the forces only in the provides sections because they define a coarse description. Better results are obtained by using smaller distances between sections with increased precision in the tip because computational results show that there is a high density of forces at the blade end. We use a preprocessing of the blade by resampling the geometrical parameters according to a finer discretization:

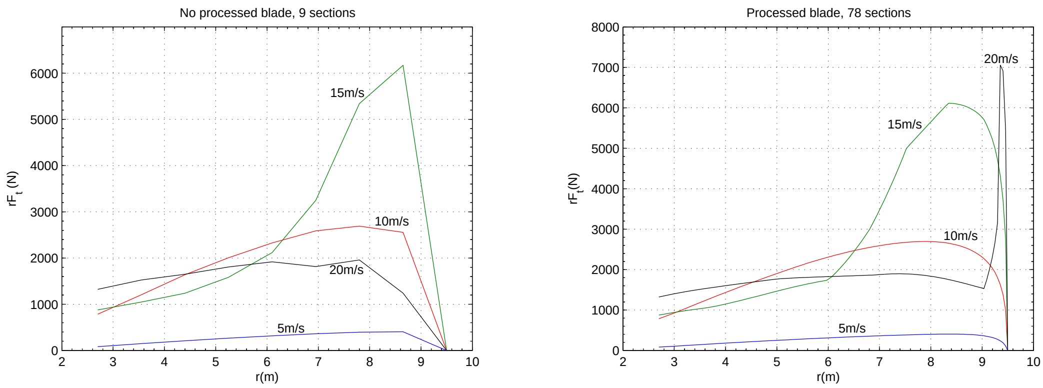

Figure 3: Predicted values for torque density, $rF_t(r)$, along the blade. At left the obtained from the provided 9 sections, whereas at right the obtained from the finer resampling with 78 sections. The coarse discretization hides important contributions of torque density for the torque and power computation.

$$r_i \Delta r_i = r_1 \Delta r_1 \qquad r_{i+1} = r_i + \Delta r_i \tag{24}$$

where Δr_1 is a constant. The discretization based on a constant value for $r_i \Delta r_i$ is used because it is the weight for torque computation $\Delta M_i = F_t(r_i) r_i \Delta r_i$. Also, computational results show a no regular distribution of torque density along the blade. Figure 3 shows the torque density $rF_t(r)$, along the blade predicted by the BEM implementation. The left subfigure shows the obtained from the provided 9 sections, whereas the right subfigure shows the obtained from the finer resampling with 78 sections, which are obtained by choosing $\Delta r_1 = 0.3m$ The coarse discretization hides important contributions of torque density to the torque and power computation.

The chord, twist and thickness in different sections are obtained by using spline approximation from the provide values. The lift and drag coefficients are known only for a discrete set of thickness values: $\{12, 15, 18, 21, 25\}$. In each section of intermediate thickness, these coefficients are interpolated from the known values by means of:

$$C_L(\alpha, r_i) = \sum_{k=1}^{5} C_{Lk}(\alpha) \Psi(t(r_i) - t_k) \tag{25}$$

where t_k are the coded values, and Ψ are interpolative functions, in this case spline functions. All thickness values in the airfoil are ranged from 12.6 to 24.6, which are into the provided airfoil family from 12 to 25.

The Figure 4 shows several computational predictions vs the experimental data for different wind speed. BEM predictions in power are very similar to experimental data in the linear zone until wind speed of 13 m/s approximately. As appointed by Lindenburg[20], differences among different tested methods in the linear, no stall, zone result from code and data mistakes, therefore the application of more complicated methods cannot improve the performances in this zone. After the linear zone, the

$V(m/s)$	$P_{exp}(Kw)$	$P_{pred}(Kw)$	$Err(\%)$
4.675	4.390	7.624	73.7
5.534	12.244	13.357	9.1
6.535	24.312	22.109	9.1
7.545	36.514	33.168	9.2
8.448	48.026	44.914	6.5
9.471	61.657	60.213	2.3
10.444	72.949	75.872	4.0
11.497	86.389	91.457	5.9
12.060	97.444	97.371	0.1
13.760	105.922	102.900	2.8
14.532	106.596	100.618	5.6
15.230	106.222	96.507	9.1
16.430	98.641	86.085	12.7

Table 1: Experimental vs predicted power curves for the Risø test turbine. The values of the relative error have two remarkable exceptions. The first at low wind speed is less significative because little power can be extracted from the wind flow at this speed. The second happens at stall zone where the BEM method, as well as most of other method, is less precise

airfoil stall generates a decreasing in the torque and power. In this zone, the BEM predictions differ from experimental data.

3 Power computation and control strategies

For a defined geometry and wind speed, the torque is depending on the parameter couple (θ, ω). This dependence is mainly related with the turbine regulation. An exact model of wind turbine regulation is complex and case dependent, because there are many posible strategies and practical implementations. The mechanical interaction between the turbine motor torque and the load resistive torque has to find an equilibrium point for a working ω. The load of the wind turbine is usually an electric generator that is connected to an electric system. It can be the grid or an isolated electric system (off-grid). The equilibrium point can be changed by modifying the pitch angle θ with two proposes. The first is to control the rotational speed ω, e.g when a synchronous generator is used. The second is to move the equilibrium point to another with greater generated power.

The simplest control strategy is for stall-regulated wind turbines with an asynchronous generator. In this case, the blades have fixed pitch and the asynchronous generator rotates at constant speed in practice. We use a simplified model for implementing control strategies to avoid overloads. This is based in what is the dynamic state of the turbine, rather than in how is achieved that state. The general case that cover several strategies is:

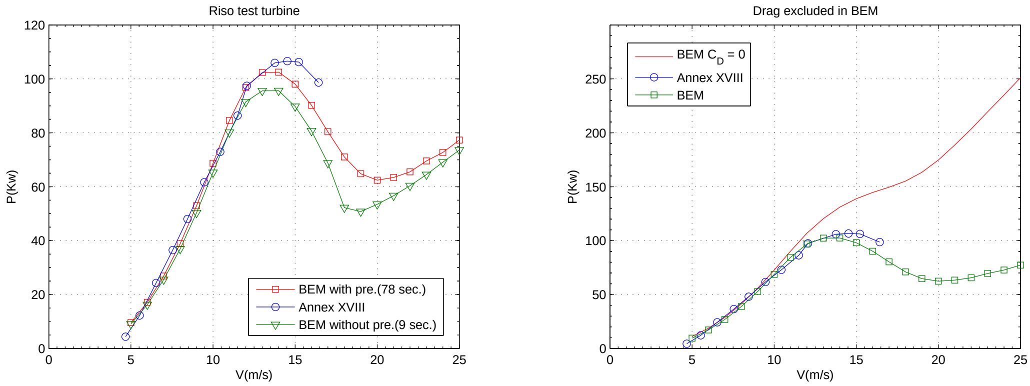

Figure 4: Comparing the power curve for the Risø test turbine with the obtained with the BEM code. High similarity in power curves are obtained in the linear zone, but some differences exit in stall zone. At the left, the differences among the experimental data provided in the Annex XVIII report an the obtained with or without resampling. At the right, the results including or excluding the drag coefficient in the induction computation. It is concluded that the drag factor, which is related with viscous friction, is very important in stall zone; it generates most of the fall in torque and power.

$$P(\mathbf{g}, V) = \max_{\omega,\theta} P(\mathbf{g}, V, \omega, \theta) \tag{26}$$

where the user must provide the range over ω and θ variables for implementing the desired control strategy. Some cases are posible. For stall-regulated turbines, a single value θ_0 must be considered for the pitch range, if also an asynchronous generator is used a single value ω_0 must be considered for rotational speed. Several control strategies can be implemented by combining fixed vs variable values for both pitch and rotational speed. The power distribution $P(\mathbf{g}, V)$ must be averaged over the wind speed probability distribution chosen for the optimization proceses:

$$P(\mathbf{g}) = \frac{\int_{V_l}^{V_u} f(V) P(\mathbf{g}, V) dV}{\int_{V_l}^{V_u} f(V) dV} = \frac{\sum_i p_i P(\mathbf{g}, V_i)}{\sum_i p_i} \tag{27}$$

where $f(V)$ is the probabilistic distribution of wind speed, usually the Weibull distribution, and $[V_l, V_u]$ is the considered wind speed range. In practice, the average value is computed from a discrete serie of wind speed values V_i and its relative probabilities p_i. The result $P(\mathbf{g})$ is only geometric dependent. This is the fitness function for the evolutionary algorithms that is proportional to the annual generated energy. The blade geometry is coded based on the chromosomal variables $\mathbf{b}$ such as $\mathbf{g}(\mathbf{b})$ is the geometry corresponding to an individual in the evolving population. The evolutionary algorithm tries to find the optimal in the space of $\mathbf{b}$ variables:

$$\mathbf{g}_{opt} = \arg\max_{\mathbf{b}} P(\mathbf{g}(\mathbf{b})) \qquad \overline{P} = P(\mathbf{g}_{opt}) \tag{28}$$

where $\mathbf{g}_{opt}$ is the optimal geometry of the wind blade and $\overline{P}$ is the maximal expected mean power.

3.1 Chromosomal coding of the turbine

Each individual in the population of the genetic algorithm is a proposal for a blade shape in chord and twist distribution. Many parameters of a turbine can be changed to improve its performance, eg. the airfoil shape or the blade number. This paper deals with the chord and twist changes because the available procedures to compute both parameters are plenty of assumptions about constant factors, eg. attack angle, lift to drag ratio and similar[2, 3]. We agree that it can be of general interest a methodology less restrictive, where the optimal distribution arises from the best performance without other a priori assumptions.

To code the chord and twist along the blade, we choose a finite number of control sections. The rest of the section values are obtained by means of polynomial interpolation. In practice, many control sections generate overfitting and increase the computational cost, and few control sections can provide a smoother distribution. We define the upper and lower limits of both chord and twist in every control section such as the value of chord and twist is codified by N_c and N_ψ bits respectively. For every control section the local values are defined as:

$$c(r) = c_l(r) + \frac{c_u(r) - c_l(r)}{2^{N_c} - 1} b_c(r) \qquad b_c(r) \in [0, 2^{N_c} - 1] \tag{29}$$

$$\psi(r) = \psi_l(r) + \frac{\psi_u(r) - \psi_l(r)}{2^{N_\psi} - 1} b_\psi(r) \qquad b_\psi(r) \in [0, 2^{N_\psi} - 1] \tag{30}$$

Several constraints must be imposed to obtain solutions with desirably properties. Some of these deal with decreasing values of twist from root to tip as well as, for structural reason, decreasing chord. To avoid *strange* individuals in the genetic populations, only those verify that $\psi(r_i) \geq \psi(r_{i+1})$ and $c(r_i) \geq c(r_{i+1})$ are valid, other ones are pruned. These constraints are imposed to all sections along the blade. With the chord property there is an additional problem, due that:

$$\frac{\partial F_t}{\partial c} > 0 \tag{31}$$

The chord improvement in any optimization procedure tends always toward increasing values. Without any additional constraint, the optimization provides always the upper limits as the optimized values. To avoid that, we can include an upper limit of the blade area because this implies a competitive decision about where the increasing in chord value is more efficient to maximice the fitness function and where the chord must be reduced.

A good practice in turbine optimization can be to improve a previous design. It can be an intermediate design in the stepwise process of aerodynamic, structural, power and control analysis. The upper limit of the area can be obtained as $(1+\delta_A)A_0$, where A_0 is the area of the original blade. With δ_A we can introduce small changes or no changes, $\delta_A = 0$, to avoid high modifications in structural performances. The chord upper and lower limits can be also obtained from the previous design as: $c(r)(1 \pm \delta_c)$, where δ_c defines the relative range of allowed changes. The evolutionary procedure can produce a chord redistribution but no global generalized increasing. By supposing a linear law between sections, the area constraint is defined as:

$$\int c(r)dr = \frac{1}{2}\sum(c_{i+1}+c_i)(r_{i+1}-r_i) \leq (1+\delta_A)A_0 \tag{32}$$

The twist optimization is different because the null value $\psi(r) = 0$ can be included in the blade. Instead of relative changes, a better practice is to allow absolute changes as a float range for twist: $\psi(r) \pm \delta_\psi$. After the optimization, a twist normalization is required because the optimal distribution do not determine the new pitch value. We have an ambiguity:

$$\theta_0 + \psi_{opt}(r) = \theta + \psi(r) \tag{33}$$

where θ_0 is the original pitch value used in the optimization. With the optimal values we must decide the value of the collective pitch and the values of the local twist distribution along r. A procedure for normalization is to define a section with null local twist, eg. at the tip $\psi(R) = 0$. In such case, the new pitch value is $\theta = \theta_0 + \psi_{opt}(R)$.

4 Results

Optimal design in wind power systems depends on the selected site[2]. The method used in the paper is: if a turbine were in a defined site, we would redesign the chord and twist of its blades to optimize its annual generated energy. Following this method, we try to optimize the Risø test turbine by modifying the chord and twist values of the LM 8.2 blade to be optimal in a site with defined Weibull distribution, eg. with parameters: $A = 8.6$ and $k = 2.66$. Several constraint are imposed to avoid that blade structural performances will be drastically changed. These are $\delta_A = 0$, that is no area increasing and $\delta_c = 0.1$, that is a top of 10% in chord value changes. With these parameters only slight chord redistribution is allowed. In twist, a more generous range is allowed: $\delta_\psi = 5^o$. Three control points are used for the chord and twist distribution; two of them are sited in the first and the last sections and the third, which controls the curvature, is sited in the middle.

The computation of the induction factor is achieved by using the successive refinement values: $a^{(n+1} = H_n(a^{(n}, a'^{(n}, \ldots)$. But there is a problem in the iterative procedure. In some cases, negative values of the attack angle, α, are required for the computation of the lift and drag coefficients, but available data are in the range from 0^o to 90^o. To solve this problem, the lift and drag tables are extended until -10^o. In the drag table, a symmetrical extension is used for negative values and a skew symmetrical in the lift one:

$$C_D(-\alpha) = C_D(\alpha) \qquad C_L(-\alpha) = 2C_L(0) - C_L(\alpha) \qquad \alpha \in [0, 10^o] \tag{34}$$

However, all the attack angle values are positive in the final solution after the convergence criterium is reached. This means that the angle extension is only used in the iterative step; it is no significative in the final solution.

The fitness function in the evolutionary optimization is the average of the power provides by BEM for wind speed from 5m/s to 20 m/s. The BEM method has been implemented in C language and embebed into Matlab by means of a MEX-file. We have tested the BEM implementation to determine the computational cost. The results show that one single evaluation of the Risø test turbine needs about 1.8ms of cpu time. The resampled test turbine was used with 78 sections. The estimated time value was obtained by averaging the cpu time taken by looping 100 times the evaluation of the turbine performance for 21 wind speed values from 5m/s to 25m/s. The processor was an 1.6Ghz Intel Pentium Mobile with 1Gb of main memory.

The Matlab genetic toolbox is used as the optimization procedure. We use default values for most of parameters of the genetic algorithm. To improve the convergency speed the option of multiple populations is used. Four populations with 50 individual in each are used with co-evolution and migration every ten generations. A fraction of the 5% on best individual migrates to other populations.

The Figure 5 and the Table 2 show the results of the optimization for twist and chord parameters of the Risø turbine. Although the LM 8.2 blade is a comercial blade for small wind turbines, already optimized, we try to find some improvements to tune it in the defined site. The optimized blade has more torsion. The normalized twist goes from 0^o at tip to 19.57^o in the first section, while the original has the range from 0^o to 15^o. Notice that the floating range of $\pm 5^o$ allows a total change of 10^o in normalized twist. However, only a change of 4.57^o is generated in the optimization. The collective pitch changes from 1.8^o to -2.986^o. Chord changes are less significative because they are strongly constrained. A slight redistribution of chord is generated with a decreasing in both external sections.

These changes in the geometrical properties generate changes in the power performance of the turbine. Figure 6 shows the power curve for both the original and the optimized turbine. The new turbine has more power for slow winds until 14.5m/s proximately and has worse performance for faster ones. However, these winds are less probable. Also, this Figure includes the graphical representation of the theoretical power distribution that is proportional to $p(V)V^3$.

The expected value for the mean power of the original turbine in the range of wind speed from 5m/s to 20m/s at the selected site is 45.474 Kw equivalent to an anual energy production of 398.35 Mwh. These values are for aerodynamics power and energy, the electrical ones are depending on the efficiency of the electric generator and the gearbox. The values for the optimized turbine are 46.685 Kw and 408.96 Mwh respectively. The optimization generates an increasing of 10.61 Mwh in the

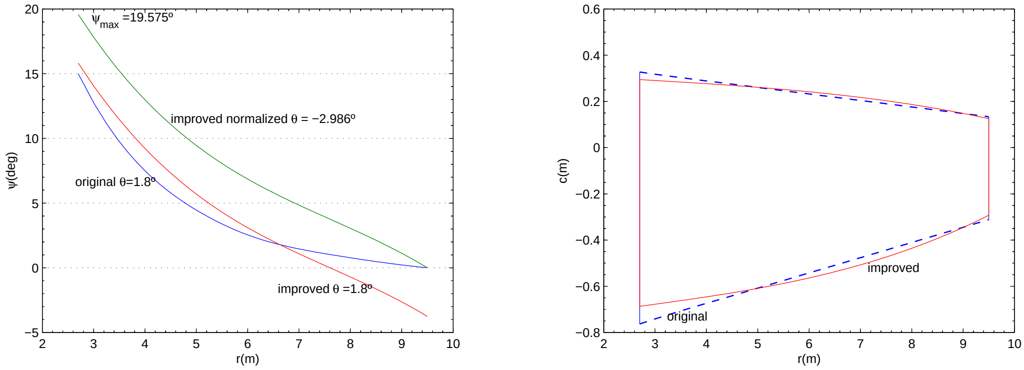

Figure 5: At left the original and optimized twist distribution. The normalized twist value is obtained with null value a tip. The new blade has more torsion that the original. Also the final blade pitch is changed from the original value of 1.8^o to -2.986^o in the optimized. At right the chord distribution graphically magnified for both the original blade, which has a trapezoidal shape, and the optimized that is more curved. The optimization generates chord reductions at both blade ends, and an increasing in the centre of the blade.

$r(m)$	$t(\%)$	$c(m)$	$\psi(deg)$	$c_{opt}(m)$	$\psi_{opt}(deg)$
2.70	24.6	1.090	15.0	0.981	19.57
3.55	20.7	1.005	9.5	0.944	14.98
4.40	18.7	0.925	6.1	0.903	11.44
5.25	17.6	0.845	3.9	0.855	8.73
6.10	16.6	0.765	2.4	0.798	6.64
6.95	15.6	0.685	1.5	0.729	4.94
7.80	14.6	0.605	0.9	0.644	3.41
8.65	13.6	0.525	0.4	0.542	1.84
9.50	12.6	0.445	0.0	0.418	0.00

Table 2: Chord and twist distribution of the original Risø test turbine and the corresponding for the optimized. The original blades has a pitch value of 1.8^o and the improved has a value of -2.986^o.

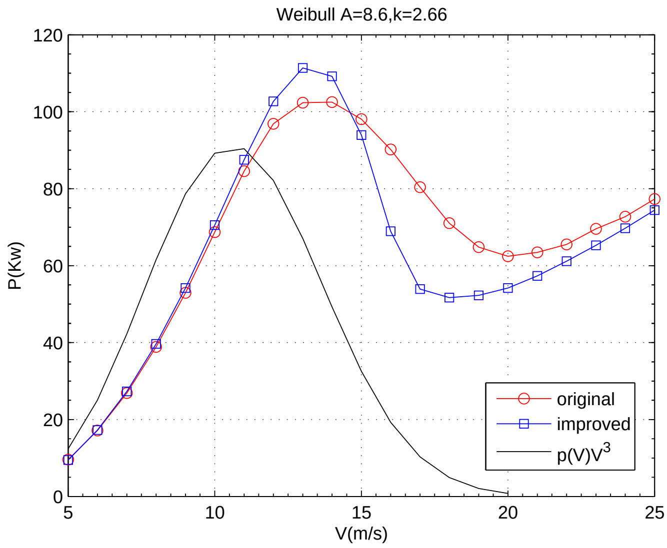

Figure 6: The power curve for the original and the site optimizad turbine. The new turbine has better performance that the original until the wind speed value of 14.5m/s proximately , and worse performance for higher values. But these values have less probability.

annual energy production that is equivalent to the 2,66% of the total.

5 Conclusion

A computational efficient methodology to obtain the chord and twist distributions for site specific blades has been presented. The method is based on the use of basic pieces of both aerodynamic technics for power prediction in wind turbines and also in optimization tools. In most of the optimization procedures, eg. gradient or genetic, the main computational load is the repetitive goal function evaluation. Therefore, we have implemented a BEM procedure that provides high quality predictions in the linear and in the near stall zones. Also, this procedure is efficient in the use of computational resources. For a test turbine, the evaluation of the forces along the blade radius took about 1.8ms. This result allows massive simulations by testing thousands of trial turbines in few seconds.

The aim of the paper is that an application for chord and twist optimization can be an useful tool to be used on the cycle design of wind turbines. The developed tool requires as input data the turbine description, the Weibull wind parameters and the allowed upper and lower limits for the chord and twist distributions. Also, the user can define an upper limit for the blade area. The procedure provides the optimal distribution for both parameters. The solution maximizes the mean annual power, which is proportional to the annual generated energy. An application of the methodology to optimize the Risø test turbine for a site has been presented. The results show that some degree of optimization is posible, but when it is applied to comercial blades already optimized the gain is no high. However, we agree to the hypothesis that the cost in refinement cycles to obtain such comercial optimized blades may be reduced. Future works will tend to involve the thickness changes similarly as the chord and twist have been optimized.

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