Planar shape representation based on Multiwavelets.

Fernando Pérez Nava[1], Antonio Falcón Martel[2]
[1] Dep. de Estadística, Inv. Op. y Computación. Universidad de La Laguna. Tenerife. SPAIN
[2] Dep. de Informática y Sistemas, Univ. de Las Palmas de Gran Canaria.Gran Canaria. SPAIN
Tel: +34 922 319184[1], +34 928 458728
e-mail: [1]fdoperez@ull.es, [2]afalcon@dis.ulpgc.es

ABSTRACT

A technique is presented to construct a multiscale representation of planar contours based on the multiwavelet transform (MWT). To generate this multiwavelet description, a partial 1-D discrete multiwavelet transform (DMWT) is applied to the vertical and horizontal components of a length-parametrized planar curve. This multiscale representation descomposes the curve into different levels of resolution and allows to reconstruct it to a desired degree of aproximation. A comparison between the multiwavelet, wavelet and the elliptic fourier transform (EFT) is presented. The results show that typical objects are well represented by a small number of multiwavelet coefficients allowing for a compact object shape representation.

Keywords: multiwavelets, shape description, multiscale methods.

1. INTRODUCTION

A powerful propery for distinguising an object from its surroundings in an image is overall shape. Shape can be used to complete the information provided by other local properties in an image such as gray level, texture or color. Therefore an efficient representation of shape information is a basic task in many areas of computer vision, video processing and analysis and computer graphics.

Many shape representations that are potentially useful for shape description have been developed [1]. Two-dimensional shape can be represented using a real or complex 1-D function. From these representations, shape descriptors can arise in the form of Chain codes, Polygonal aproximations, Elliptic fourier descriptors, B-Splines or Multiscale gaussian descriptions.

Recently it was presented in [2] a multiscale descriptor of planar shapes based on periodized wavelet analysis in the continous metric space $L^2([0,1])$. In [3] an analysis of such description was presented based on the Discrete Wavelet Transform. It was shown that shapes can be represented with a small approximation error, allowing for an efficient shape representation.

In this work we extend this analysis to the multiwavelet case. Multiwavelets extend wavelets and their main motivation is that they can simultaneously posses desirable properties such as symmetry, orthogonality, shorter support for a given approximation order, which are not possible for any real-valued scalar wavelet [4]. A comparative between both coding approaches is presented.

2. THE MULTIWAVELET DESCRIPTION.

A multiple wavelet basis uses translations and dilations of L≥2 scaling functions $\phi_1,\dots,\phi_L$ and L wavelet functions $\varphi_1,\dots,\varphi_L$. Assuming L=2 a 1-D function f can be expresed as:

where:

$$f(x)=\sum_{k\in Z} C_{J,k}^{\mathrm{T}}\, 2^{\frac{J}{2}}\,\mathbf{\Phi}(2^J x-k)+\sum_{j=1}^{J}\sum_{k\in Z} D_{J,k}^{\mathrm{T}}\, 2^{\frac{J}{2}}\,\mathbf{\Psi}(2^J x-k)$$

Let then $\mathbf{r}(s)$ = (x(s), y(s)) be a discrete

$$\mathbf{\Phi}(x)=(\phi_1(x),\phi_2(x))^{\mathrm{T}},\ \mathbf{\Psi}(x)=(\psi_1(x),\psi_2(x))^{\mathrm{T}},$$
$$C_{j,k}=(c_{j,k,1},c_{j,k,2})^{\mathrm{T}},\ D_{j,k}=(d_{j,k,1},d_{j,k,2})^{\mathrm{T}}$$

parametrized closed planar curve that represents the shape of an object of interest. If the multiwavelet transform is applied independently to each of the x(s), y(s) functions, we can describe the planar curve in terms of a decomposition of $\mathbf{r}(s)$:

$$\mathbf{r}(s)=\sum_{k\in Z}\mathbf{C}_{J,k}\, 2^{\frac{J}{2}}\,\mathbf{\Phi}(2^J s-k)+\sum_{j=1}^{J}\sum_{k\in Z}\mathbf{D}_{J,k}\, 2^{\frac{J}{2}}\,\mathbf{\Psi}(2^J s-k)$$

$$\mathbf{C}_{j,k} = \begin{pmatrix} C^{\mathrm{T}}{}_{j,k;\mathrm{x}} \\ C^{\mathrm{T}}{}_{j,k;\mathrm{y}} \end{pmatrix}, \quad \mathbf{D}_{j,k} = \begin{pmatrix} D^{\mathrm{T}}{}_{j,k;\mathrm{x}} \\ D^{\mathrm{T}}{}_{j,k;\mathrm{y}} \end{pmatrix}.$$

where subindex x and y represent function pertenence.

obtaining a multiresolution representation of shape (Fig 1):

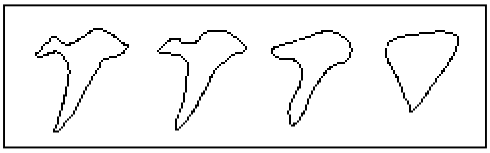

Fig. 1 Multiwavelet representation of shape

2.1 Properties of the multiwavelet descriptors

Both wavelets and multiwavelets share the capability of detecting and representing local features. The MWDT has invariance, uniqueness and stability properties asuming that the parametrization of curves has the same starting point . This is a consequence of a well known fact: the WDT and MWDT coefficients are not invariant to parametrization shifts. In order to make the tests, an arbitrary point of the contour was fixed and then multiwavelet, wavelet and fourier descriptions.

3. EXPERIMENTAL ANALYSIS

The MWDT was efficiently implemented using the multiwavelet pyramid algorithm. As test images, we used a set of synthetic planar curves (see Fig 2). For each image we only used the outline of the figure, interior contours were not processed. All contours were resampled in order to obtain 256 points and decomposition was taken to the coarsest level. Then the set of the n most important coefficients (coefficients with maximum amplitude) were obtained for different values of n=32, 64, 128.

The mean distance error between the reconstructed curve curve from this set of coefficients and the original curve is then show for distinct wavelet families: Haar, Daubechies class D4 and least asymmetric LA8, and distinct multiwavelet families (Geronimo-Hardin-Massopust GHM, Chu-Lian CL, and Shen-Tan-Tham SA(4)) for comparison purposes the Elliptic Fourier Transform is also included.

	Fourier	Haar	D4	La8	GMH	CL	SA(4)
n= 32	4.0	6.1	2.8	2.4	2.5	2.7	2.3
n= 64	1.8	3.3	1.2	1.0	1.1	1.1	1.0
n=128	0.8	1.7	0.6	0.4	0.5	0.5	0.5

Table 1 Mean distance error for test images.

Numerical results show in general, that best compresion is achieved with the SA(4) multiwavelet, with results similar to the LA8 wavelet. After these, best results are obtained with the GMH and CL multiwavelet, D4 wavelet, Fourier descriptors and Haar wavelets. In Figure 3 it is shown the mean error distance (n=32) for all test data for Fourier elliptic descriptors (thick line), LA8 wavelet (discontinous line) and SA(4) multiwavelet (thin line). Test contours are numbered left to right and top to bottom. From this figure we can see how the SA(4) multiwavelet obtains sightly superior results to the LA8 wavelet. The worst contour for both SA(4) and LA(8) is show in Figure 4 where the original contour (thick line), is shown with the LA8 wavelet approximation (discontinous line) and SA(4) multiwavelet approximation (thin line).

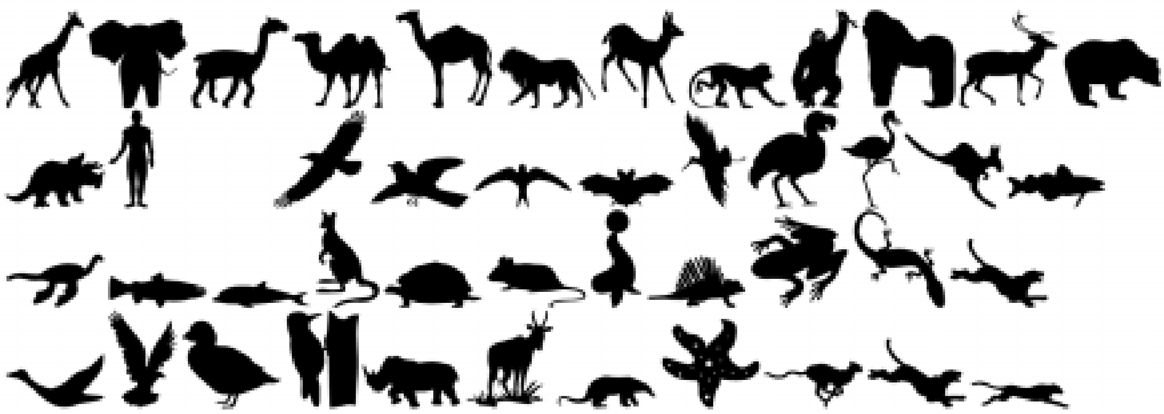

Figure 2 Test images

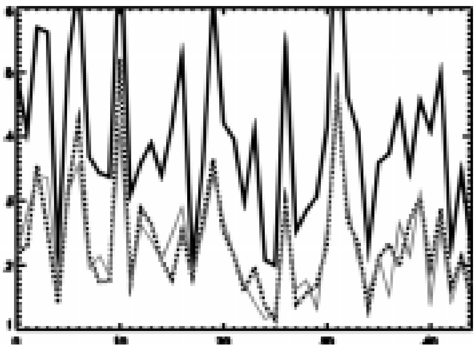

Figure 3 Error comparison

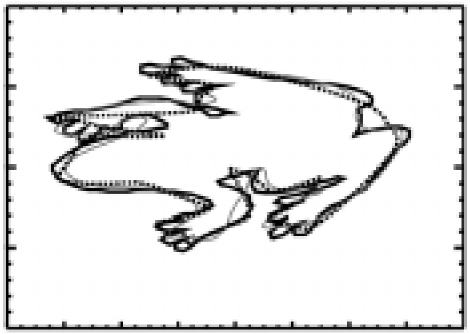

Figure 4 Maximum test image error

In shape processing it is generally desirable a not only a description with a small mean distance. Often it is useful to obtain a small maximum distance between the original contour and its approximation. To test the capacity of this description to obtain a low maximum error, the 95% percentile of the maximum distance was computed for the LA8 and SA(4) multiwavelet for all images in the database obtaining the results in Table 2.

	n= 32	**n =64**	**n =128**
LA8 wavelet	4.14	1.72	0.76
SA(4) multiwavelet	4.09	1.80	0.79

Table 2 The 95% percentile of the distance error

Results show how the error for both descriptions is similar, in general however the SA(4) description produces smoother and symmetric representation and therefore it is visually more appealing.

3. CONCLUSIONS

A comparison between wavelet and multiwavelet families for shape description has been shown. Results show that the SA(4) wavelet and LA8 multiwavelet produce similiar and the best results of all families. However the SA(4) family produces a description visually more atractive.

4. REFERENCES

[1] S. Loncavic, A Survey of Shape Analysis Techniques, Patter Recognition, Vol 31, nº8, pp 983-1001(1998)

[2] G.C.-H. Chuang and C.-C.J. Kuo, “Wavelet description of planar curves”, IEEE Trans. on Image Procesing, Vol 5, Nº 1, pp 56-70, (1996)

[3] D. Gatica-Pérez and F. García Ugalde, “Compact Representation of planar curves based on wavelet shape descriptors for multimedia applications”, Vision Interface 99 pp- 604-611 (1999)

[4] G.Plonka and V. Strela, From Wavelets to Multiwavelets, Math. Methods for Curves and Surfaces, Vanderbilt Univ. Press, pg-375-399 (1998).