CONDENSATION-BASED CONTOUR TRACKING WITH SOBOLEV SMOOTHNESS PRIORS

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ABSTRACT: This paper proposes a combination of contour deformation modelling in Sobolev spaces and the Condensation filter to track an object over a sequence of images. As Sobolev spaces are smoothness spaces this allows to control the smoothness of the contour deformation extending previous wavelet representations. We also introduce a probabilistic model for the wavelet deformation of the contour that induces a prior distribution for contour deformation. The deformation model is used to generate an stochastic dynamic model for contour evolution in time. Computational results are presented that show applications of this formulation.

Keywords: Active Contours, Deformation modelling, Wavelets, Sobolev spaces, Contour tracking, Condensation filter.

1 INTRODUCTION

Deformable models (McInerney & Terzopoulos, 1996) are object models that possess shape-varying capability, which makes them suitable for representing non-rigid objects. A deformable model can be described by a vector of parameters that span a multidimensional space. Usually deformations are confined in a region of the whole space and prior information about preferred deformations of the model can be obtained. This preference is then expressed in terms of a deformation energy that penalizes some deformations. The importance of this prior information is that it simplifies and increases the robustness of the solutions to some problems like fitting and tracking.

The deformation energy is coupled with a data mismatch criterion that measures the degree of discrepancy between the model and some measures extracted from an image. Model matching can then be formulated as an optimization problem of a combined criterion function that is defined in terms of both the deformation and the mismatch energy.

An important subset of deformable models are active contour models where an object is characterized by its external shape. Several parametric representations have been used to represent deformable contours from parametric b-spline representations like the b-snake (Menet, Saint Marc & Medioni, 1990) to elliptic Fourier descriptors (Staib & Duncan, 1992). We will use wavelet based representations (Chuang & Kuo, 1996). They possess several properties that makes them attractive like locality, multiresolution, energy compaction and decorrelation.

In this paper new probabilistic prior models for deformation modeling are presented. These models are based on a wavelet representation of shape and Sobolev smoothness spaces (Choi & Baraniuk, 1999). We also present experimental results for the contour tracking problem in which the Condensation filter has been employed.

This paper is divided in seven parts: in Section 2 a wavelet based deformation model is presented and its relation with Sobolev spaces is established. In Section 3 a prior probabilistic distribution for contour deformation is introduced. In Section 4 we express the deformation models in terms of linear shape space. To study the tracking problem in Section 5 wavelet based dynamic models are presented and these models are applied in Section 6. Finally in Section 7 we present the conclusions of this paper.

2 WAVELET BASED REPRESENTATIONS OF SHAPE

Deformation models for wavelet representations of shape are related to functional smoothness spaces. In this paper, Sobolev spaces will be used in which the smoothness of the deformation can be parametrically specified.

2.1 Wavelet Shape Representation

A wavelet basis uses translations and dilations of a scaling function ϕ and a wavelet function φ. If a curve $\boldsymbol{r}(u)$ is closed and parameter u belong to an interval I=[0,L] then we obtain the representation:

$$\boldsymbol{r}(u) = \boldsymbol{c}_{0,0}\varphi_{0,0}(u) + \sum_{\substack{j\geq 0 \\ 0\leq l<2^j}} \boldsymbol{d}_{j,l}\psi_{j,l}(u) \quad \textbf{(1)}$$

with the coefficients in the curve expansion defined as:

$$\boldsymbol{c}_{0,0} = \left(\left\langle x(u), \varphi_{0,0}(u)\right\rangle, \left\langle y(u), \varphi_{0,0}(u)\right\rangle\right)^{\mathrm{T}}, \quad \boldsymbol{d}_{j,l} = \left(\left\langle x(u), \psi_{j,l}(u)\right\rangle, \left\langle y(u), \psi_{j,l}(u)\right\rangle\right)^{\mathrm{T}} \quad \textbf{(2)}$$

where: $\langle f, g \rangle = \frac{1}{L}\int_0^L f(u)\, g(u)du$, and basis and wavelet functions are normalized so that:

$\|\varphi_{0,0}\|_{L_2(I)} = \|\psi_{j,l}\|_{L_2(I)} = 1$ and norm is defined as: $\|f\|^2_{L_2(I)} = \langle f, f \rangle$

In practice, we will work with finite number of coefficients in the representation:

$$\boldsymbol{r}(u) = \boldsymbol{c}_{0,0}\varphi_{0,0}(u) + \sum_{\substack{0\le j<J \\ 0\le l<2^j}} \boldsymbol{d}_{j,l}\psi_{j,l}(u) \tag{3}$$

obtaining a curve with $2N$ degrees of freedom ($N = 2^J$)

Curve expansion can be concisely written in matrix form. First we express scaling and wavelet functions in a vector as:

$$\boldsymbol{G}_W(u) = (\varphi_{0,0}(u) \quad \psi_{0,0}(u) \quad \dots \quad \psi_{J-1,2^{J-1}-1}(u))^{\mathrm{T}} \tag{4}$$

with scaling and wavelet coefficients in vectors $\boldsymbol{w}^x$ and $\boldsymbol{w}^y$ defined as:

$$\begin{aligned} \boldsymbol{w}^x &= (c_{0,0;x} \quad d_{0,0;x} \quad \dots \quad d_{J-1,2^{J-1}-1;x})^{\mathrm{T}} \\ \boldsymbol{w}^y &= (c_{0,0;y} \quad d_{0,0;y} \quad \dots \quad d_{J-1,2^{J-1}-1;y})^{\mathrm{T}} \end{aligned} \tag{5}$$

Then to describe curve $\boldsymbol{r}$(u) in matrix form we define:

$$\boldsymbol{F}_W(u) = \boldsymbol{I}_2 \otimes \boldsymbol{G}_W(u)^{\mathrm{T}} = \begin{pmatrix} \boldsymbol{G}_W(u)^{\mathrm{T}} & \boldsymbol{0}_N^{\mathrm{T}} \\ \boldsymbol{0}_N^{\mathrm{T}} & \boldsymbol{G}_W(u)^{\mathrm{T}} \end{pmatrix} \tag{6}$$

where $\otimes$ denotes the Kronecker product and $\boldsymbol{0}_N$ a null vector of N components. Then we may write:

$$\boldsymbol{r}(u) = \boldsymbol{F}_W(u)\boldsymbol{w}, \qquad \boldsymbol{w} = (\boldsymbol{w}^{x\,\mathrm{T}}, \boldsymbol{w}^{y\,\mathrm{T}})^{\mathrm{T}} \tag{7}$$

2.2 Wavelet based Modeling of Curve Deformation

In order to control the smoothness of deformations we will consider Sobolev spaces $W^{\alpha}(L_2(I))$, $0 < \alpha < \infty$. These spaces have, very roughly speaking, "α derivatives in $L_2(I)$". Norms in Sobolev spaces (Wojtaszczyk, 1997) can be extended from real functions to curves defining a curve norm in $W^{\alpha}(L_2(I))\times W^{\alpha}(L_2(I))$ as follows:

$$\|\boldsymbol{r}\|_{W^{\alpha}(L_2(I))} \equiv \sqrt{|c_{0,0;x}|^2 + |c_{0,0;y}|^2} + \left(\sum_{j\ge 0} \sum_{k=0}^{2^j-1} 2^{2\alpha j} \left(|d_{j,k;x}|^2 + |d_{j,k;y}|^2 \right) \right)^{1/2} \tag{8}$$

3 WAVELET PROBABILISTIC DEFORMATION MODEL IN SOBOLEV SPACES.

The simplest wavelet transform statistical models are obtained assuming that the coefficients in the curve representation are independent. Under the independence assumption, modeling reduces to simply specifying the marginal distribution of each wavelet coefficient.

It can be shown that the gaussian distribution is related to Sobolev spaces by means of the following theorem that extends the result in (Choi & Baraniuk, 1999):

Theorem

Let a curve $\mathbf{C} \equiv \boldsymbol{r}(u)= (x(u), y(u))^{\mathrm{T}}$ be decomposed on its wavelet representation. Denote by $\boldsymbol{I}_2$ the identity matrix of order 2 and suppose that vectors of the wavelet representation $\boldsymbol{d}_{j,k}$ are independently and identically distributed as:

$$\boldsymbol{d}_{j,k} = (d_{j,k;x}, d_{j,k;y})^{\mathrm{T}} \sim N(\mathbf{0}_2, \sigma_j^2 \boldsymbol{I}_2), \qquad \sigma_j = 2^{-j\beta}\sigma_0,\ \beta > 0 \text{ and } \sigma_0 > 0. \tag{9}$$

Then the realizations of the model are almost surely in the curve extension of the Sobolev Space $W^{\alpha}(L_2(I))$ if and only if $\beta > \alpha + 1/2$.

To complete the definition of the model we have to specify the distribution for the coefficient associated with the scaling function $\boldsymbol{c}_{0,0}$. This coefficient is associated with a translation of shape and we will assume that it is normally distributed and independent of the non-translation components $\boldsymbol{d}_{j,k}$. This obviously does not change the pertenence of the curve to its Sobolev Space. Then:

$$\boldsymbol{c}_{0,0} = \begin{pmatrix} c_{0,0,x} \\ c_{0,0,y} \end{pmatrix} \sim \boldsymbol{N}(\mathbf{0}_2, \sigma_{TR}^2 \boldsymbol{I}_2) \tag{10}$$

The wavelet probabilistic deformation model in (**9**) and (**10**) can be expressed in matrix form as:

$$\mathrm{P}(\boldsymbol{r}) \equiv \mathrm{P}(\boldsymbol{w}) \propto \exp\left(-\frac{1}{2}\boldsymbol{w}^{\mathrm{T}}\boldsymbol{S}_w \boldsymbol{w}\right), \quad \boldsymbol{r} = \boldsymbol{F}_W \boldsymbol{w} \quad \text{with:} \tag{11}$$

$$\boldsymbol{\Sigma} = \boldsymbol{S}_w^{-1} = \boldsymbol{I}_2 \otimes \begin{pmatrix} \sigma_{TR}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{DEF}^2 2^{-2\beta 0} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \sigma_{DEF}^2 2^{-2\beta(J-1)} \end{pmatrix}$$

As shown before, parameter β is related to the smoothness of the deformation. It can be shown that parameters σ_{TR}^2 and σ_{DEF}^2 are related to the uncertainty of the contour. If a curve $\mathbf{C} \equiv \boldsymbol{r}(u)= (x(u), y(u))^{\mathrm{T}}$ is decomposed on its wavelet representation and its deformations are given by the wavelet probabilistic model shown in (**11**), then the mean square displacement $\overline{\rho}^2$ verifies:

$$\overline{\rho}^2 = \mathrm{Trace}(\boldsymbol{\Sigma}) = \overline{\rho}_{TR}^2 + \overline{\rho}_{DEF}^2, \quad \overline{\rho}_{TR}^2 = 2\sigma_{TR}^2, \quad \overline{\rho}_{DEF}^2 = 2\sigma_{DEF}^2 \frac{1-N^{-2\beta+1}}{1-2^{-2\beta+1}}, \beta > 1/2 \tag{12}$$

where N is the number of wavelet coefficients in the decomposition of the parametric functions as defined in (**3**) and $\overline{\rho}_{TR}^2$ and $\overline{\rho}_{DEF}^2$ are the mean square displacement due to translation and non-translation.

4. WAVELET BASED DEFORMATION MODELS IN SHAPE SPACES

In certain applications it is possible to reduce the number of degrees of freedom in the model constraining its deformations. A linear shape-space $L(\boldsymbol{H}, \overline{\boldsymbol{w}})$ (Blake, Curwen & Zisserman, 1993) is a linear mapping of a "shape space vector" $\boldsymbol{x} \in \mathbb{R}^{N_X}$ to a wavelet vector $\boldsymbol{w} \in \mathbb{R}^{N_W}$:

$$\boldsymbol{w} = \boldsymbol{H}\boldsymbol{x} + \overline{\boldsymbol{w}} \qquad \textbf{(13)}$$

where $\boldsymbol{H}$ is a $N_W \times N_X$ "shape matrix" that will be assumed of full column rank. Vector $\overline{\boldsymbol{w}}$ represents a base curve against which shape variations are measured. Several shape spaces can be constructed: from geometric shape spaces or key frames shape space to learned shape spaces derived from the PCA transform.

The probabilistic model defined in Section 3 for wavelet space induces a probabilistic model in shape space:

$$\mathrm{P}(\boldsymbol{x}) \propto \exp\left(-\frac{1}{2}\boldsymbol{x}^T \boldsymbol{S}_{\boldsymbol{x}}\ \boldsymbol{x}\right), \quad \boldsymbol{S}_{\boldsymbol{x}} = \boldsymbol{H}^{\mathrm{T}} \boldsymbol{S}_w \boldsymbol{H} \qquad \textbf{(14)}$$

As shown in (**12**) mean square displacement is related to the trace of the covariance matrix. Therefore if a curve $\mathbf{C} \equiv \boldsymbol{r}(u) = (x(u), y(u))^{\mathrm{T}}$ is decomposed on its wavelet representation and its deformations are given by $\boldsymbol{H}\boldsymbol{x} + \overline{\boldsymbol{w}}$ with $\boldsymbol{x} \sim \boldsymbol{N}(\boldsymbol{0}_{N_X}, \boldsymbol{S}_{\boldsymbol{x}}^{-1})$, then the mean square displacement $\overline{\rho}^2$ around the contour can be expressed in terms of the oblique projection matrix :

$$\boldsymbol{P}_{\boldsymbol{S}_w} = \boldsymbol{H}\boldsymbol{H}_{\boldsymbol{S}_w}^{+}, \quad \boldsymbol{H}_{\boldsymbol{S}_w}^{+} = \left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{S}_w\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{S}_w \qquad \textbf{(15)}$$

as:

$$\overline{\rho}^2 = \mathrm{Trace}\,(\boldsymbol{P}_{\boldsymbol{S}_w}\boldsymbol{\Sigma}) \qquad \textbf{(16)}$$

Sometimes it may be desirable that the prior term influences the shape of the curve but not its position or orientation. To achieve the desired invariance over a subspace S_{INV} with oblique projection matrix $\boldsymbol{P}_{\boldsymbol{S}_w}^{INV}$ contained in shape space, the information matrix $\boldsymbol{S}_{\boldsymbol{x}}$ in (**14**) is set to:

$$\boldsymbol{S}_{INV_x^-} = \boldsymbol{G}_{INV^-}^{\mathrm{T}}\boldsymbol{S}_x\boldsymbol{G}_{INV^-} \qquad \textbf{(17)}$$

through matrix $\boldsymbol{G}_{INV^-}$ that projects $\boldsymbol{x}$ in the complement S_{INV^-} of subspace S_{INV} by means of projection matrix $\boldsymbol{P}_{\boldsymbol{S}_w}^{INV^-}$, defined as:

$$G_{INV^-} = H_{S_w}^{+} P_{S_w}^{INV^-} H \quad \text{with} \quad P_{S_w}^{INV^-} = (I_{2N} - P_{S_w}^{INV}) \qquad \textbf{(18)}$$

5 WAVELET-BASED DYNAMIC MODELS IN SOBOLEV SPACES

To obtain a wavelet based solution of the tracking problem, dynamic models for wavelets that define contour evolution in time are presented. The prior deformation model presented has to be extended to deal with the problem of tracking the curve over a sequence of images. In order describe shape motion a second order autoregressive AR(2) process will be used:

$$\boldsymbol{w}_t - \overline{\boldsymbol{w}} = \boldsymbol{A}_1 (\boldsymbol{w}_{t-1} - \overline{\boldsymbol{w}}) + \boldsymbol{A}_2 (\boldsymbol{w}_{t-2} - \overline{\boldsymbol{w}}) + \boldsymbol{B}\boldsymbol{n}_t \qquad \textbf{(19)}$$

where $\boldsymbol{A}_1$, $\boldsymbol{A}_2$, $\boldsymbol{B}$, are square matrices of order N_W and $\boldsymbol{n}_t$ is a vector normally distributed of mean $\boldsymbol{0}_{N_W}$ and covariance matrix equal to the identity.

The wavelet model leads to the model in shape space:

$$\boldsymbol{x}_t - \overline{\boldsymbol{x}} = \boldsymbol{A}_1 (\boldsymbol{x}_{t-1} - \overline{\boldsymbol{x}}) + \boldsymbol{A}_2 (\boldsymbol{x}_{t-2} - \overline{\boldsymbol{x}}) + \boldsymbol{B}\boldsymbol{n}_t \qquad \textbf{(20)}$$

where now $\boldsymbol{A}_1$, $\boldsymbol{A}_2$, $\boldsymbol{B}$, are square matrices of order N_X and $\boldsymbol{n}_t$ is a vector normally distributed of mean $\boldsymbol{0}_{N_X}$ and covariance matrix equal to the identity.

5.1 Partitioned harmonic motion between subspaces

If we have a several orthogonal subspaces S_i with associated matrix in shape space $\boldsymbol{G}_i$ defined as in (**18**) motion can be decomposed in each subspace (Blake & Isard, 1998) defining:

$$\boldsymbol{A}_1 = \sum_i a_{1,i} \boldsymbol{G}_i, \quad \boldsymbol{A}_2 = \sum_i a_{2,i} \boldsymbol{G}_i, \quad \boldsymbol{B} = \sum_i b_i \, \boldsymbol{G}_i \boldsymbol{S}_x^{1/2}, \quad a_{1,i}, a_{2,i}, b_i \in \mathbb{R}$$

where $a_2 = -\exp(-2\,\delta\,\tau)$, $a_1 = 2\exp(-\delta\,\tau)\cos(2\pi f\,\tau)$ **(21)**

Parameter τ represents the sample interval, δ represents the damping rate and f the frequency of oscillation.

This allows the motion of the contour to be expressed in terms of the motion on each subspace. If we define:

$$a_i^2 = \frac{(1 - a_{2,i})}{(1 + a_{2,i})(1 - a_{1,i} - a_{2,i})(1 + a_{1,i} - a_{2,i})} \qquad \textbf{(22)}$$

then the mean vector $\boldsymbol{E}_{\infty i}$ and covariance matrix $\boldsymbol{C}_{\infty i}$ for the limit distribution of the autoregressive model in subspace S_i verifies:

$$\boldsymbol{E}_{\infty i} = \boldsymbol{G}_i \overline{\boldsymbol{x}}, \qquad \textbf{(23)}$$
$$\boldsymbol{C}_{\infty i} = b_i^2 a_i^2 \boldsymbol{G}_i \boldsymbol{S}_{\boldsymbol{x}}^{-1} \boldsymbol{G}_i^{\mathrm{T}}$$

The total mean square displacement $\overline{\rho}^2$ can be obtained from the mean square displacement of each model $\overline{\rho}_i^2$ as:

$$\overline{\rho}^2 = \sum_i \overline{\rho}_i^2 \tag{24}$$

Then to obtain a mean displacement $\overline{\rho}_i$ we must set b_i to:

$$b_i = \frac{\overline{\rho}_i}{a_i \sqrt{\text{Trace}(\boldsymbol{P}^i_{S_w} \boldsymbol{\Sigma})}} \tag{25}$$

so parameters related to the stochastic part of the model can be defined in terms of the deterministic parameters.

5.2 Constant velocity model

A subclass of harmonic motion is the constant velocity model in which $\delta = f = 0$. In this case there is no steady state and $\overline{\rho}_i^2$ can not be used to characterize the stochastic component. Then it can be shown that the mean square displacement $\overline{\rho}^2(t)$ grows asymptotically as:

$$\overline{\rho}(t) \approx \gamma \ t^{3/2}, \quad b = \frac{\gamma}{\sqrt{\text{Trace}(\boldsymbol{P}_{S_w} \boldsymbol{\Sigma})/3}} \tau^{3/2} \tag{26}$$

6 THE TRACKING PROBLEM

In this section the results obtained from the wavelet based dynamical models are used in the contour tracking problem. To define the image observation model note that in one dimension observations reduce to a set of scalar positions $\boldsymbol{Z}=(d_1,d_{2,\ldots,}d_m)$ and the observation density has the form $P(\boldsymbol{Z} \mid x)$ where x is an scalar position. Then a density function can be defined as (Isard, 1998):

$$p(\boldsymbol{Z} \mid x) \propto 1 + \frac{1}{\sqrt{2\pi}\sigma\alpha} \sum_m e^{-\frac{(d_m - x)^2}{2\sigma^2}} \tag{27}$$

where σ represents the uncertainty in position of the scalar positions d_i and α balances the distribution in case that no one of the d_i in vector $\boldsymbol{Z}$ corresponds to x. In two dimensions the probability distribution is constructed as the product of one dimensional densities, evaluated independently along curve normals. The use of a non-gaussian density for the observation model poses the problem into the Condensation filter framework (Isard 1998). In this framework conditional densities are approximated through a set Q_t of samples $\hat{\boldsymbol{q}}_t^{(i)}$ weighted with discrete probabilities $\pi_{t-1}^{(i)}$. When the AR(2) model is used we have:

$$Q_{t-1}=\{\hat{\boldsymbol{q}}^{(i)}_{t-1},\pi^{(i)}_{t-1}, i=1\ldots N_Q\}, \quad \hat{\boldsymbol{q}}^{(i)}_{t-1}=(\hat{\boldsymbol{x}}^{(i)}_{t-1},\hat{\boldsymbol{x}}^{(i)}_{t-2}) \qquad \textbf{(28)}$$

Then this sample set evolves in time using the evolution equations of the Condensation filter. These equations are applied in two steps:

1.- Prediction

Approximate the prior distribution generating new elements $\hat{\boldsymbol{q}}'^{(i)}_{t-1}=(\hat{\boldsymbol{x}}'^{(i)}_{t-1},\hat{\boldsymbol{x}}'^{(i)}_{t-2})$, i=1..N_Q sampling with replacement from Q_{t-1}

Evaluate a shape space prediction with the AR(2) dynamic model:

$$\hat{\boldsymbol{x}}^{-(i)}_{t} - \bar{\boldsymbol{x}} = \boldsymbol{A_1}\,(\hat{\boldsymbol{x}}'^{(i)}_{t-1} - \bar{\boldsymbol{x}}) + \boldsymbol{A_2}\,(\hat{\boldsymbol{x}}'^{(i)}_{t-2} - \bar{\boldsymbol{x}}) + \boldsymbol{S}_{\boldsymbol{x}}^{-1/2}\,\boldsymbol{n}^{(i)}_{t}, \qquad \hat{\boldsymbol{x}}^{-(i)}_{t-1} = \hat{\boldsymbol{x}}'^{(i)}_{t-1} \qquad \textbf{(29)}$$

where $\boldsymbol{n}^{(i)}_{t}$ is a sample from a zero mean identity covariance normal distribution.

2.- Measure

For every sample evaluate the adjustment probability:

$$\pi^{(i)}_{t} = p(\boldsymbol{Z}_t \mid \boldsymbol{x}_t = \hat{\boldsymbol{x}}^{-(i)}_{t}) \text{ and normalize so that } \sum_i \pi^{(i)}_{k} = 1 \qquad \textbf{(30)}$$

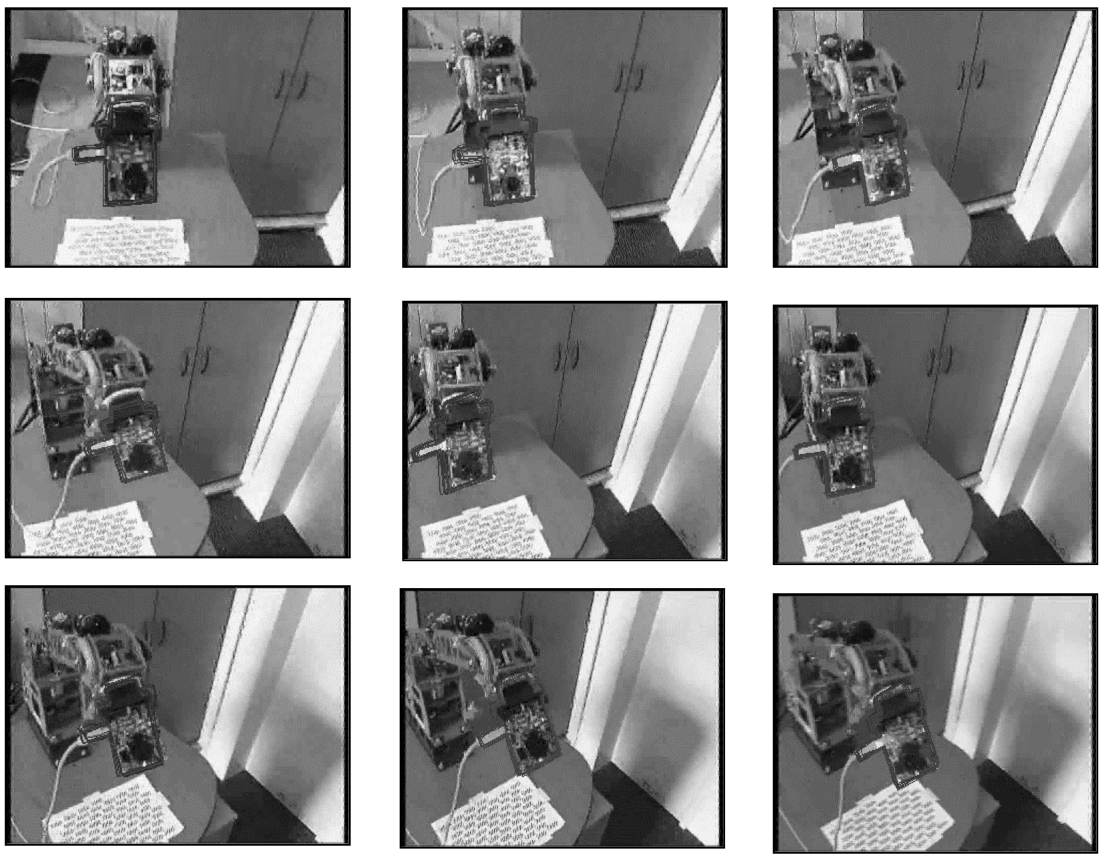
In Figure 1 (see below) we show an example of tracking with the Condensation filter. The problem to be solved is to follow the outline of a robotic head in a laboratory setting. There are several sources that may distract the tracker due to the borders found intro the head and the borders in the background. Change of position and deformation of the head has two principal components translation and affine change. We expect the robotic head to move freely across the image and to have moderate affine deformations. The dynamic mode used is the partitioned harmonic motion between subspaces. The parameters have been:

Translation	δ= 0	f=0	γ=20
Affine	δ=5	f=0	$\bar{\rho}$ =5

Table 1 Dynamical parameters on each subspace for the robotic head example

Figure 1. Tracking with the Condensation filter. Images are numbered 1-9 top to bottom and left to right

The election of the parameters show that no oscillatory movement is expected. Translational movement is defined with the constant velocity model allowing a free movement through the image and a moderate velocity in the affine subspace is supposed where mean square displacement is set to 25 pixels.



The uncertainty of the measures has been set to σ_D=2 pixels and the deformation parameter in the Sobolev space has been estimated to β=1.7. The wavelet function has been Daubechies D_{12}.

To show the results of the tracking process best 10 members of a population of 100 in set Q_t are superimposed in the image. It can be seen how the tracking algorithm follows with precision the head despite its motion and the movement of the camera. Notice by seeing images 1 and 9 of the sequence that the mouse movement has a strong affine component that can be determined through the model.

7. CONCLUSIONS

A new model for contour deformations using wavelets has been presented. It relates contour deformations to Sobolev smoothness spaces. This allows different degrees of smoothness to be enforced without altering the balance of uncertainty between prior deformation model and data extracted from the image. The deformation framework is expressed in terms of linear shape space models combining deformations from the smoothness model with geometric deformations. This deformation model is extended to a dynamic model that can be used to solve the tracking problem. Experimental results are shown for the tracking problem with the Condensation filter in real images and results are discussed.

ACKNOWLEDGMENTS

This research has been partially supported by the Canary Islands Government DGUI under project PI2000/042

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